The Higgs transverse momentum

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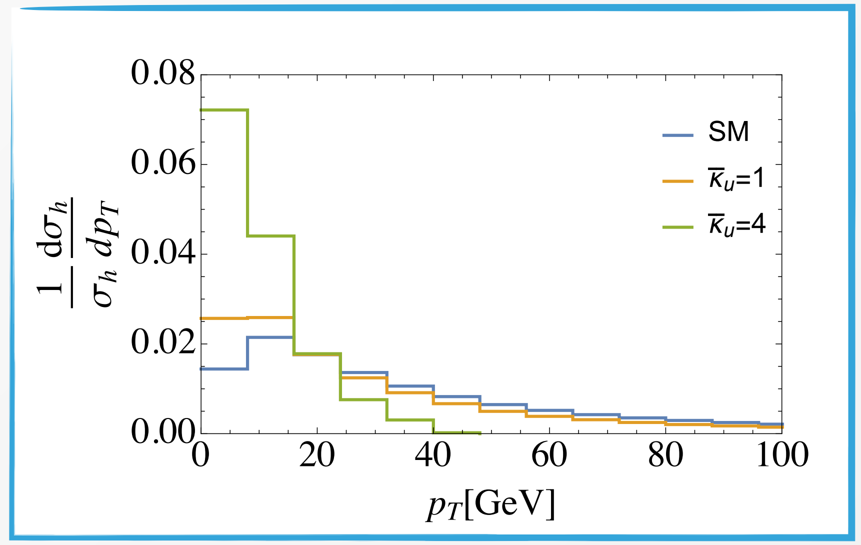
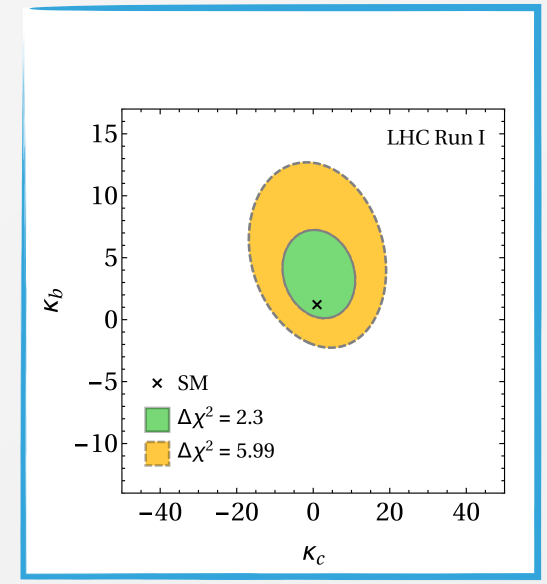
Why bother?

- ~40 inverse femtobarns collected in 2016
- Increase in statistics enables study of **differential distributions** in detail
- Transverse momentum distribution of the Higgs boson is sensitive to **new physics**

Light Yukawa

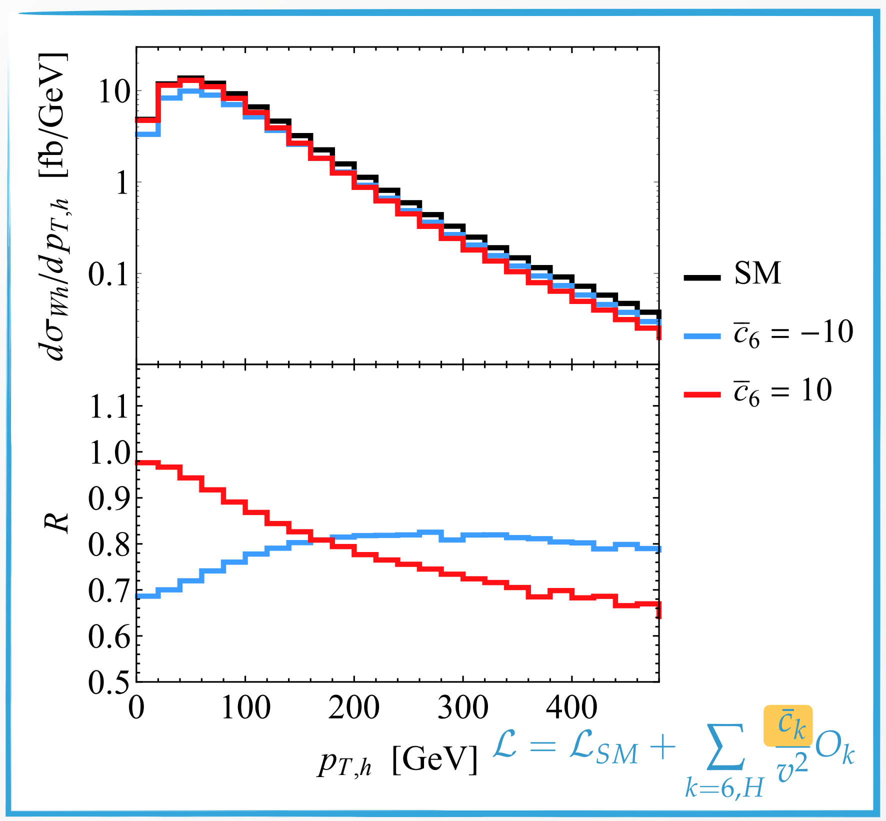


[Soreq et al, 1606.09621]

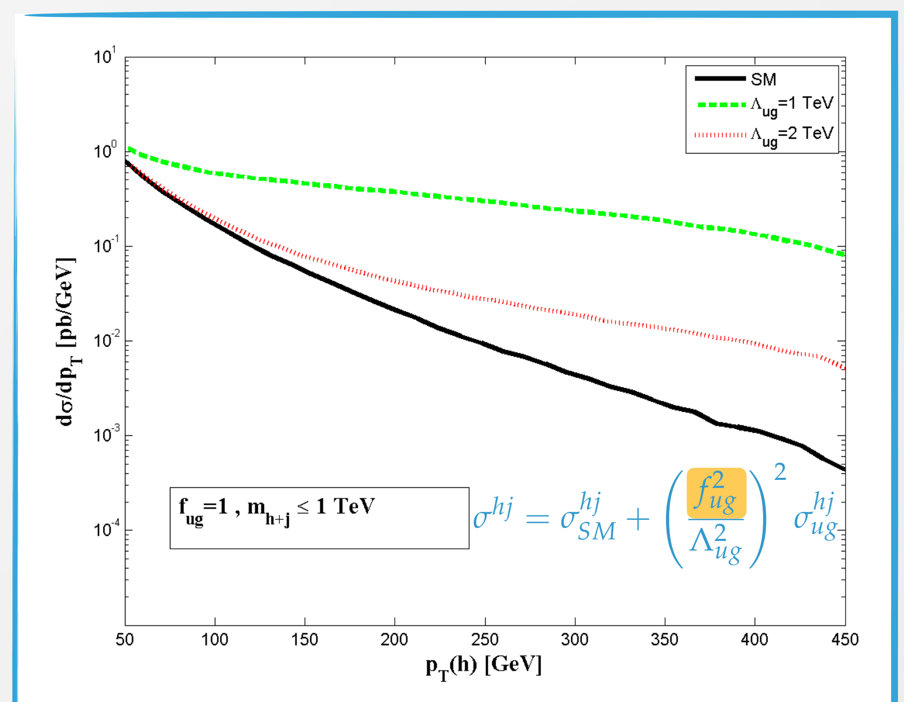


[Bishara et al.,1606.09253]

Trilinear coupling



[Bizon et al.,1610.05771]



[Cohen et al., 1705.09295]

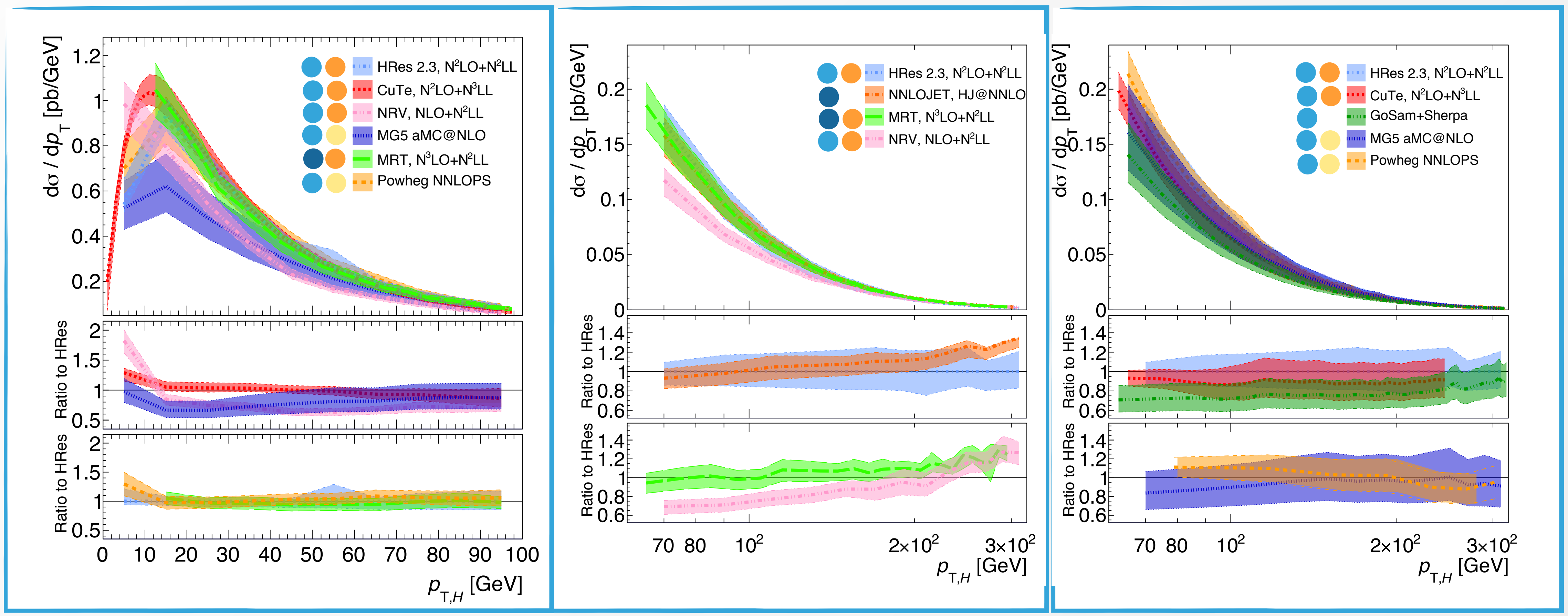
Theoretical prelude: Yellow Report 2016

α_s^3 α_s^4 α_s^5

formal FO accuracy

LL NLL NNLL N3LL

formal RES accuracy



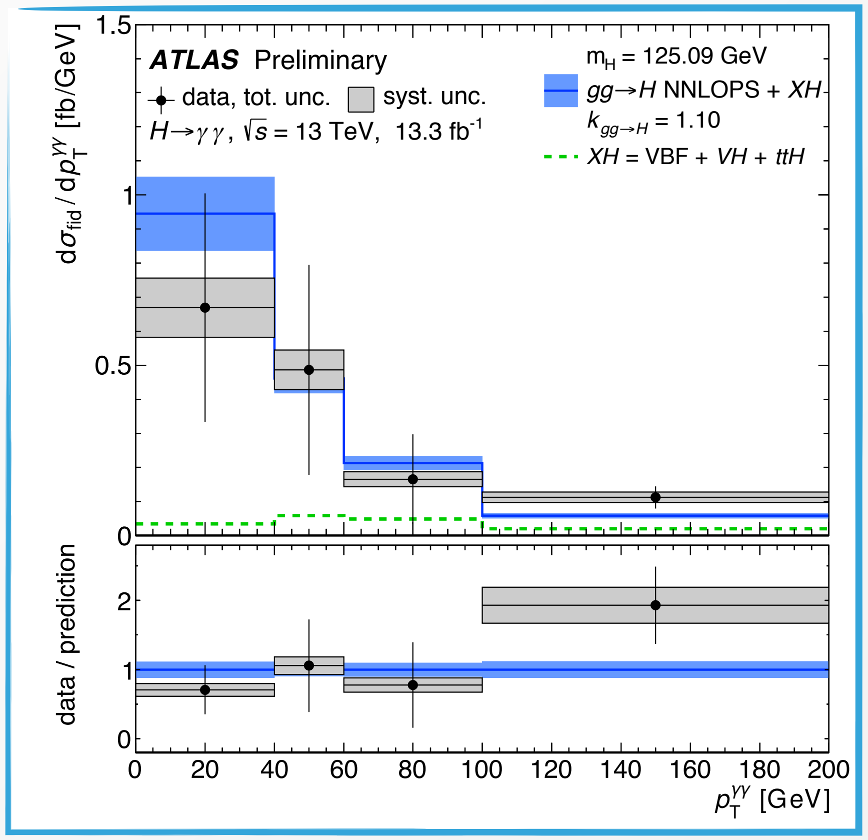
not all predictions include the same set of "uncertainties"

(all include QCD scale variations)

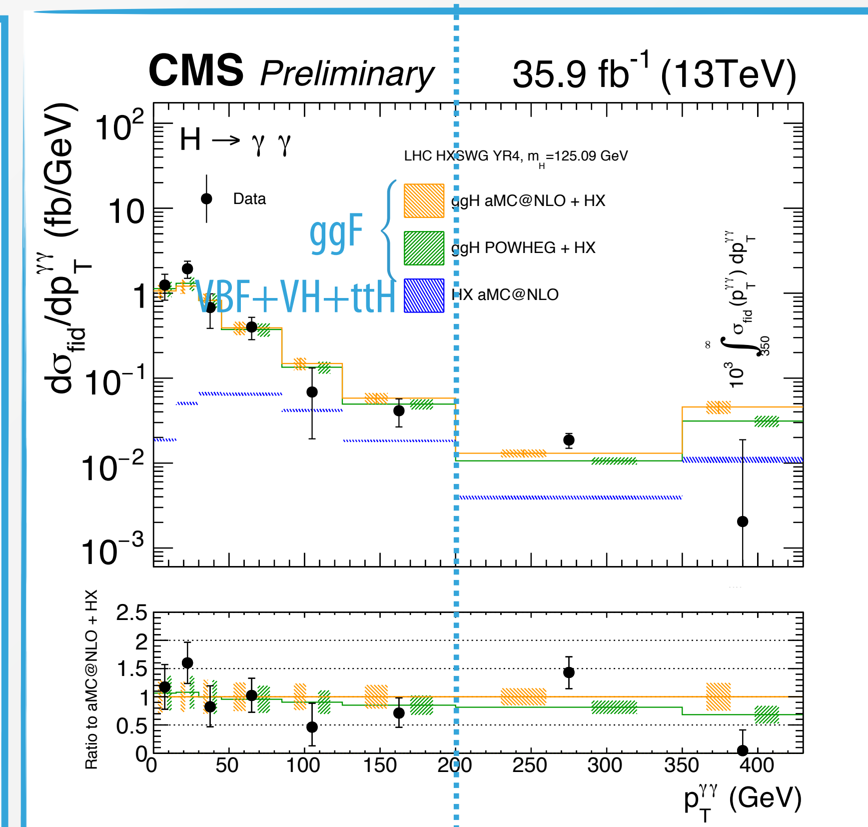
Experimental prelude: Run II results

H → γγ

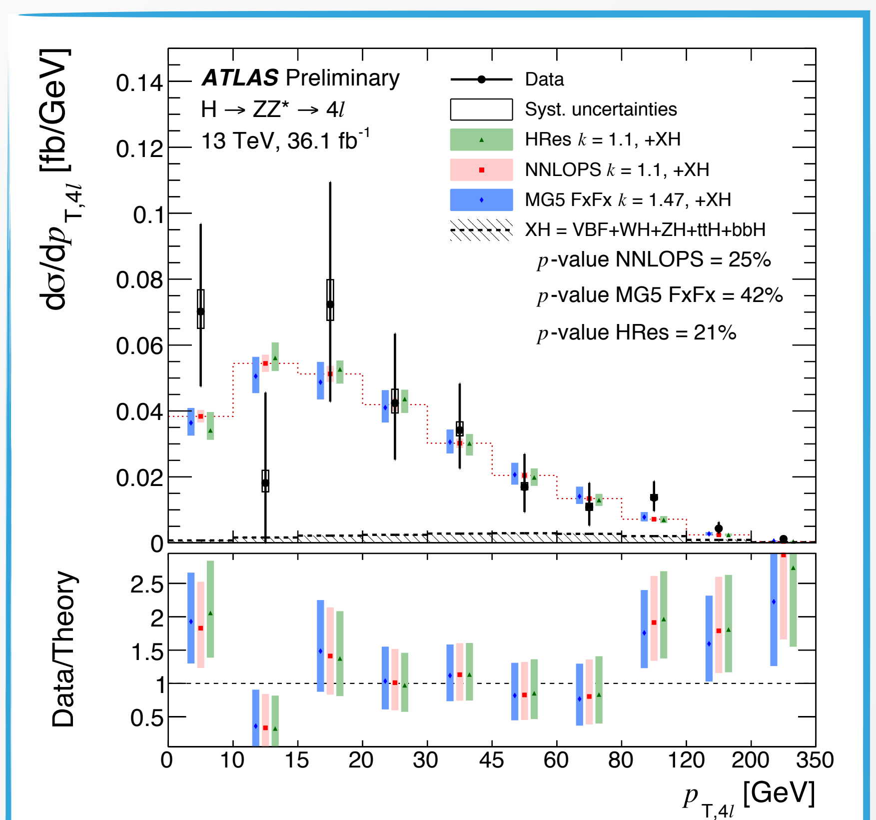
- precise reconstruction of the diphoton invariant mass
- Signal fitted in **each differential bin**
- Good agreement with Standard Model predictions



ATLAS-CONF-2016-067



CMS PAS HIG-17-015



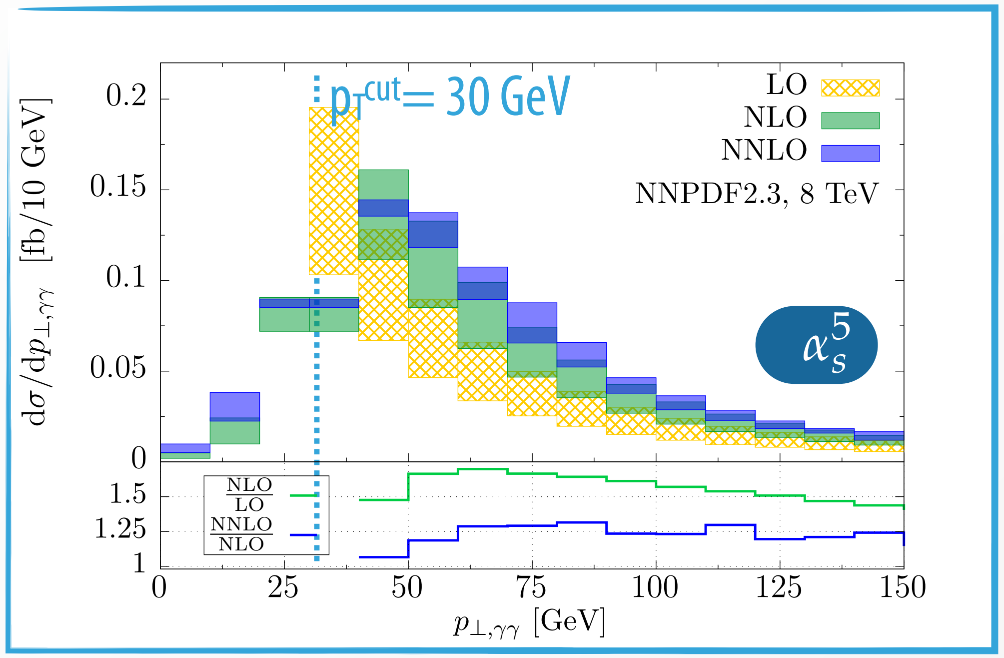
ATLAS-CONF-2017-032

H → 4l

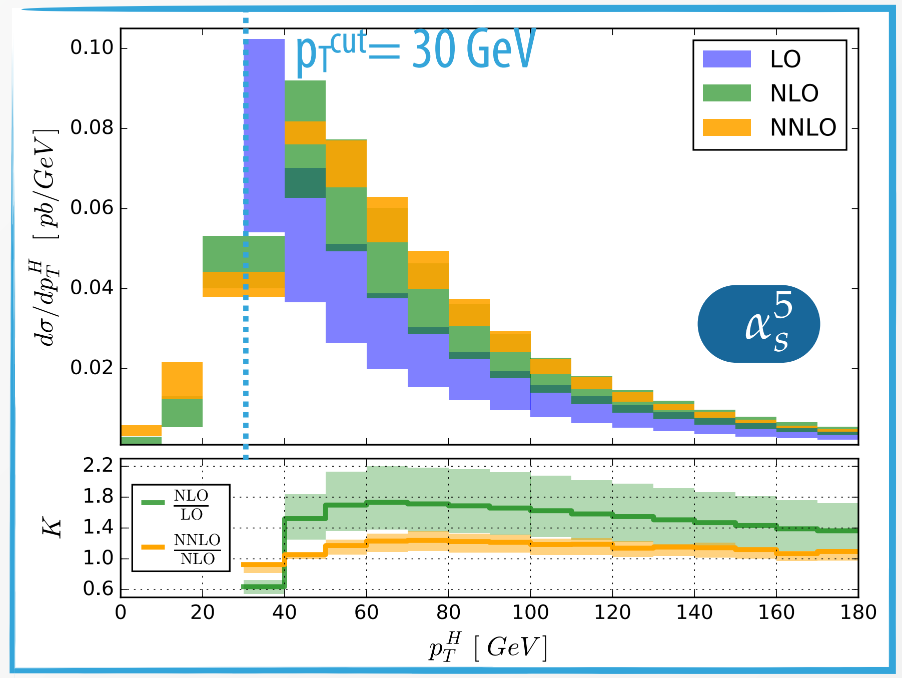
- measured cross sections at high slightly higher than the predictions
- Distribution is consistent with the (rescaled) SM predictions within the uncertainties

Fixed-order predictions: state-of-the-art

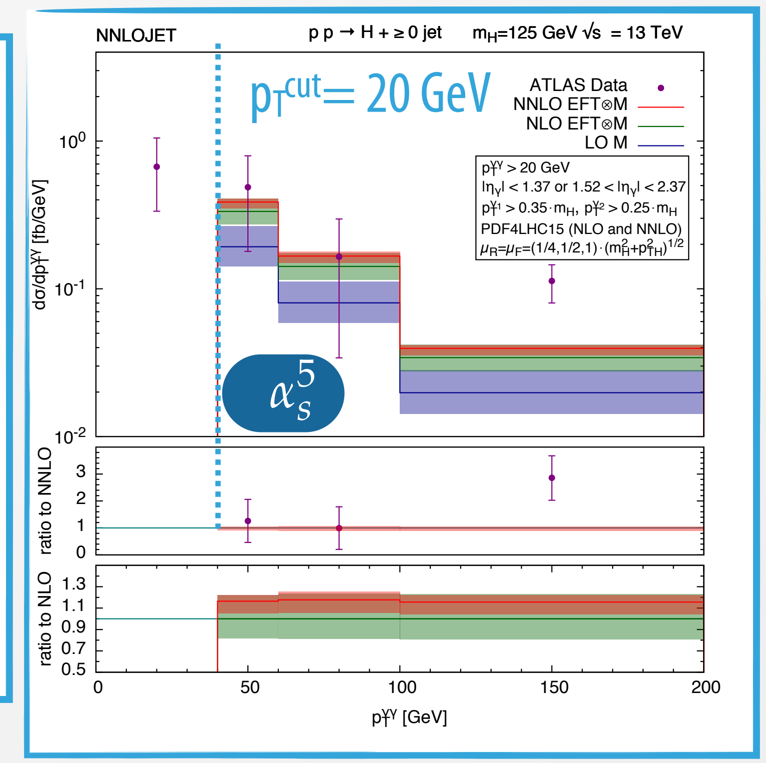
Fixed-order predictions available through NNLO QCD in the EFT
NNLO correction ~ 10-20%



[Boughezal et al. 1504.07922]
[Caola et al.1508.02684]



[Boughezal et al. 1505.03893]



[Chen et al. 1607.08817]

- **sector**- improved residue **subtraction** approach
- fiducial cross sections

- **jettiness subtraction**

- **antenna subtraction**
- comparison with ATLAS data

Fixed Scale Choice

$$\mu = m_H$$

Dynamical Scale Choice

$$\mu = \frac{1}{2}\sqrt{m_H^2 + (p_T^H)^2}$$

Resummation in the small-p_T region

Resummation

Fixed-order results are crucial to obtain reliable theoretical predictions away from the **soft** and **collinear** regions of the phase space

However, regions dominated by soft and collinear QCD radiation affected by **large logarithms**

$$\frac{1}{p_T}\alpha_s^n \ln^k(p_T/M), \qquad k \leq 2n-1$$

Perturbative series spoiled → All-order **resummation** of the logarithmically enhanced terms

Effects propagate away from the singularity, **resummation is necessary** to obtain a good control of the small-p_T region

$$\Sigma(v) = \int_0^v \frac{d\sigma}{dv'}dv' \sim e^{\alpha_s^n L^{n+1} + \alpha_s^n L^n + \alpha_s^n L^{n-1} + \dots} \qquad v = p_T/M$$

LL NLL NNLL

Logarithmic counting commonly defined at the level of the logarithm of the integrated cross section

Zeros in the small-p_T region and b-space formulation

Two different mechanisms give a contribution in the small p_T region

- configurations where the transverse momenta of the radiated partons is small (**Sudakov limit**) — Exponential suppression — Sudakov peak region
- configurations where p_T tends to zero because of cancellations of non-zero transverse momenta of the emissions (**azimuthal cancellations**) — Power suppression $\Sigma \sim \mathcal{O}(p_T^2)$ — $p_T \to 0$ limit

Power-law scaling at very small p_T

For inclusive observables the vectorial nature of the cancellations can be handled via a **Fourier transform**

[Parisi, Petronzio '78; Collins, Soper, Sterman '85]

[Catani, Grazzini '11][Catani et al. '12,Gehrmann][Luebbert, Yang '14]

coefficient functions

$$\frac{d^2\Sigma(v)}{d\Phi_B dp_t} = \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1c_2}}{d\Phi_B} \int b\, db\, p_t J_0(p_t b)\, \mathbf{f}^T(b_0/b)\, \mathbf{C}^{c_1;T}_{N_1}(\alpha_s(b_0/b)) H_{\rm CSS}(M)\, \mathbf{C}^{c_2}_{N_2}(\alpha_s(b_0/b))\mathbf{f}(b_0/b)$$

hard-virtual corrections

$$\times \exp\left\{-\sum_{\ell=1}^{2}\int_0^M \frac{dk_t}{k_t}\mathbf{R}'_{{\rm CSS},\ell}(k_t)\,\Theta(k_t - \frac{b_0}{b})\right\}$$

$$R_{\rm CSS}(b) = \sum_{l=1}^{2}\int_{b0/b}^{M}\frac{dk_T}{k_T}R'_{{\rm CSS},l}(k_T) = \sum_{l=1}^{2}\int_{b_0/b}^{M}\frac{dk_T}{k_T}\left(A_{{\rm CSS},\ell}(\alpha_s(k_T))\ln\frac{M^2}{k_T^2} + B_{{\rm CSS},\ell}(\alpha_s(k_T))\right)$$

anomalous dimensions

[Davies, Stirling '84] [De Florian, Grazzini '01] [Becher, Neubert '10][Li, Zhu '16][Vladimirov '16]

Momentum space

[Monni, Re, Torrielli, Phys.Rev.Lett. 116 (2016) no.24, 242001]
[Bizon, Monni, Re, LR, Torrielli, 1705.09127]
[Ebert, Tackmann 1611.08610] talk by Markus

Is it possible to obtain a formulation in momentum space?

Not possible to find a closed analytic expression in direct space which is both a) free of logarithmically subleading corrections and b) free of singularities at finite p_T values [Frixione, Nason, Ridolfi '98]

Why? A naive logarithmic counting at small p_T is not sensible, as one loses the correct power-suppressed scaling if only logarithms are retained: it's not possible to reproduce a power behaviour with logs of p_T/M

(logarithms of b do not correspond to logarithms of p_T)

Necessary to establish a well defined logarithmic counting in momentum space in order to reproduce the correct behaviour of the observable at small p_T

Since b-space formulation works well, why should one bother so much for a single observable?

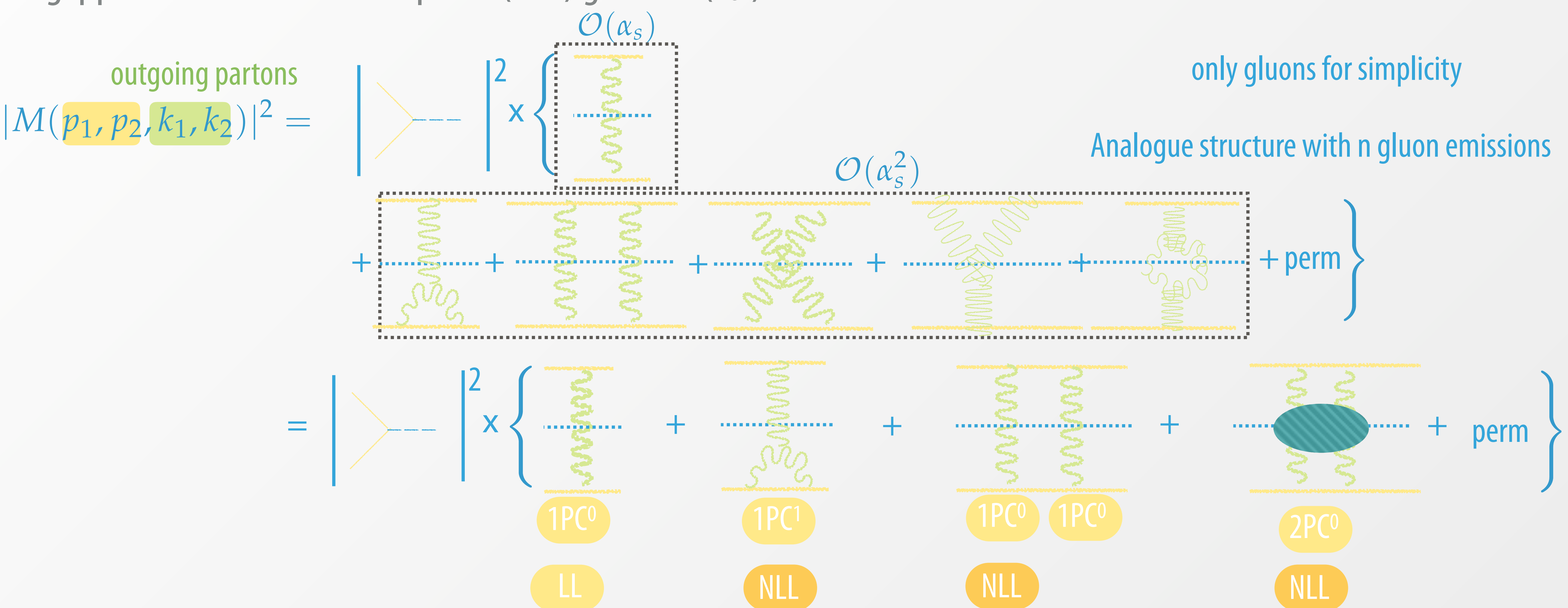
- No need to have a factorization theorem established (more **observable independent** than b-space formulation)
- Important to understand the dynamics of the radiation to improve generators
- What we learn will have a broader application range, possible generalisation beyond the simple inclusive-observable case
- Possibility to perform **joint resummation** of observables
- As a byproduct, the result in momentum space can be implemented in a code fully differential in the Born phase space (easy to introduce cuts, dynamical scales, etc)

Logarithmic counting

[Monni, Re, Torrielli, Phys.Rev.Lett. 116 (2016) no.24, 242001]
[Bizon, Monni, Re, LR, Torrielli, 1705.09127]

Necessary to establish a **well defined logarithmic counting**: possibile to do that by decomposing the squared amplitude in terms of n-particle correlated blocks (**nPC**)

e.g. pp → H + emission of up to 2 (soft) gluons $O(\alpha_s^2)$



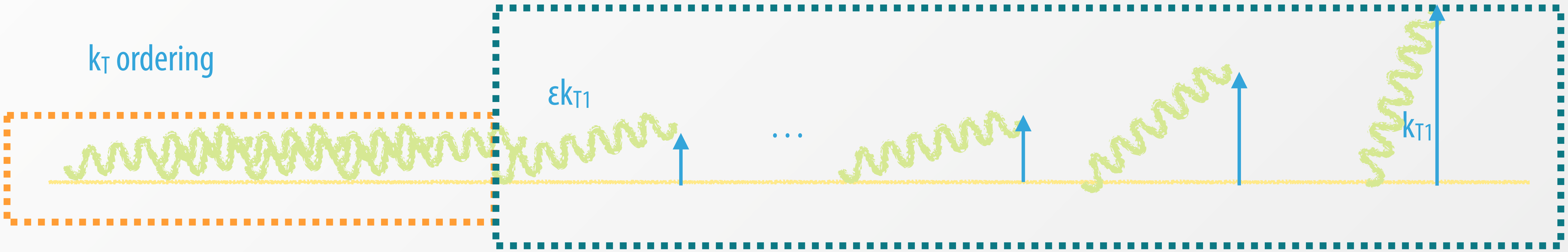
Logarithmic counting defined in terms of **nPC blocks** (owing to rIRC safety of the observable)

Resolved and unresolved emissions

For inclusive observables (such as Higgs p_T) $V(p_1, p_2, k_1, \ldots, k_n) = V(p_1, p_2, k_1 + \cdots + k_n)$

$$|M(p_1, p_2, k_1, \ldots, k_n)|^2 = |M_B(p_1, p_2)|^2 \times \frac{1}{n!} \left\{ \prod_{i=1}^{n} \left(\underbrace{|M(k_i)|^2}_{\text{1PC}} + \underbrace{\int [dk_a][dk_b] |\tilde{M}(k_a, k_b)|^2}_{\text{2PC}} \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} - \vec{k}_{ti}) \delta(Y_{ab} - Y_i) + \underbrace{\int [dk_a][dk_b][dk_c] |\tilde{M}(k_a, k_b, k_c)|^2}_{\text{3PC}} \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} + \vec{k}_{tc} - \vec{k}_{ti}) \delta(Y_{abc} - Y_i) + \ldots \right) \right\}$$

Introduction of a **resolution scale** εk_{T1}



unresolved emission

resolved emission

can be integrated inclusively to cancel the divergences of the virtuals (rIRC): exponential factor

$$e^{-R(\varepsilon k_{t1})}$$

Sudakov form factor

ε dependence cancels against the resolved real corrections

treated exclusively: for inclusive observables can be parametrised exactly as a Sudakov **unintegrated** in k_t and azimuthal angle

Momentum space formulation

need some care in the treatment of the hard-collinear emissions

Result can be expressed as

$$\frac{d\Sigma(v)}{d\Phi_B} = \int_{\mathcal{C}_1} \frac{dN_1}{2\pi i} \int_{\mathcal{C}_2} \frac{dN_2}{2\pi i} x_1^{-N_1} x_2^{-N_2} \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1c_2}}{d\Phi_B} \mathbf{f}^T_{N_1}(\mu_0) \hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) \mathbf{f}_{N_2}(\mu_0)$$

DGLAP anomalous dimensions

RG evolution of coefficient functions

$$\hat{\mathbf{\Sigma}}^{c_1,c_2}_{N_1,N_2}(v) = \left[\mathbf{C}^{c_1;T}_{N_1}(\alpha_s(\mu_0)) H(\mu_R) \mathbf{C}^{c_2}_{N_2}(\alpha_s(\mu_0))\right] \int_0^M \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi}$$
$$\times e^{-\mathbf{R}(\epsilon k_{t1})} \exp\left\{-\sum_{\ell=1}^{2}\left(\int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \frac{\alpha_s(k_t)}{\pi} \mathbf{\Gamma}_{N_\ell}(\alpha_s(k_t)) + \int_{\epsilon k_{t1}}^{\mu_0} \frac{dk_t}{k_t} \mathbf{\Gamma}^{(C)}_{N_\ell}(\alpha_s(k_t))\right)\right\}$$
$$\sum_{\ell_1=1}^{2}\left(\mathbf{R}'_{\ell_1}(k_{t1}) + \frac{\alpha_s(k_{t1})}{\pi} \mathbf{\Gamma}_{N_{\ell_1}}(\alpha_s(k_{t1})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_1}}(\alpha_s(k_{t1}))\right)$$
$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \sum_{\ell_i=1}^{2}\left(\mathbf{R}'_{\ell_i}(k_{ti}) + \frac{\alpha_s(k_{ti})}{\pi} \mathbf{\Gamma}_{N_{\ell_i}}(\alpha_s(k_{ti})) + \mathbf{\Gamma}^{(C)}_{N_{\ell_i}}(\alpha_s(k_{ti}))\right)$$
$$\times \Theta\left(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1})\right)$$

Result valid for all inclusive observables (e.g. p_T, φ^*)

$$V(k) = d_l g_l(\phi) \frac{k_T}{M}$$

unresolved emission + virtual corrections

resolved emission

Formulation **equivalent to b-space** result (up to a scheme change in the anomalous dimensions)

$$\frac{d^2\Sigma(v)}{d\Phi_B dp_t} = \sum_{c_1,c_2} \frac{d|M_B|^2_{c_1c_2}}{d\Phi_B} \int b\, db\, p_t J_0(p_t b)\, \mathbf{f}^T(b_0/b) \mathbf{C}^{c_1;T}_{N_1}(\alpha_s(b_0/b)) H(M) \mathbf{C}^{c_2}_{N_2}(\alpha_s(b_0/b)) \mathbf{f}(b_0/b)$$
$$\times \exp\left\{-\sum_{\ell=1}^{2} \int_0^M \frac{dk_t}{k_t} \mathbf{R}'_\ell(k_t)\left(1 - J_0(bk_t)\right)\right\}$$

$$(1 - J_0(bk_t)) \simeq \Theta(k_t - \frac{b_0}{b}) + \frac{\zeta_3}{12} \frac{\partial^3}{\partial \ln(Mb/b_0)^3} \Theta(k_t - \frac{b_0}{b})$$

Resummation in momentum space

Formulation in Mellin space already implementable. However, it is convenient to perform the evaluation entirely in momentum space

In previous formula, resummation of logarithms of $k_{T,i}/M$

subleading logarithms in p_T

free of singularity at low p_T values
(power-law scaling)

$k_{Ti}/k_{T1} \sim O(1)$
(everywhere in the resolved phase space, due to rIRC safety)

Integrands can be expanded about $k_{Ti} \sim k_{T1}$ to the desired accuracy: more efficient

Sudakov region: $k_{T1} \sim p_T$

$\ln(M/p_T)$ resummed at the desired accuracy

+ additional subleading terms that **cannot be neglected**

azimuthal region: $k_{Ti} \sim k_{T1}$

correct description of the kinematics after expansion $k_{Ti} \sim k_{T1}$

correct scaling of the cumulant **$O(p_T^2)$**

Result at NLL accuracy

The divergences cancel with the terms contained in the resolved real radiation

$$= e^{-R'(k_{T1}) \ln \frac{1}{\epsilon}} \qquad R' = \frac{d}{d\ln(M/k_{t1})} R$$

we expanded around k_{T1}

$$\frac{d\Sigma(v)}{d\Phi_B} = \int \frac{dk_{t1}}{k_{t1}} \frac{d\phi_1}{2\pi} \partial_L \left(-e^{-R(k_{t1})} \mathcal{L}_{\rm NLL}(k_{t1}) \right) \epsilon^{R'(k_{t1})} \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^{1} \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'(k_{t1}) \Theta(v - V(\{\tilde{p}\}, k_1, \ldots, k_{n+1}))$$

resolved emission

$$\zeta_i = k_{ti}/k_{t1}$$

parton luminosity at NLL reads

$$\mathcal{L}_{\rm NLL}(k_{t1}) = \sum_{c,c'} \frac{d|M_B|^2_{cc'}}{d\Phi_B} f_c(k_{t1}, x_1) \, f_{c'}(k_{t1}, x_2)$$

At higher logarithmic accuracy, it includes coefficient functions and hard-virtual corrections

This formula can be evaluated by means of fast Monte Carlo methods

RadISH (Radiation off Initial State Hadrons)

Result at N^{3}LL accuracy

$$
\begin{aligned}
\frac{d\Sigma(v)}{d\Phi_B} &= \int \frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}\partial_L\left(-e^{-R(k_{t1})}\mathcal{L}_{\mathrm{N^3LL}}(k_{t1})\right)\int d\mathcal{Z}[\{R',k_i\}]\Theta\left(v - V(\{\tilde{p}\},k_1,\dots,k_{n+1})\right)\\
&+\int \frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}e^{-R(k_{t1})}\int d\mathcal{Z}[\{R',k_i\}]\int_0^1\frac{d\zeta_s}{\zeta_s}\frac{d\phi_s}{2\pi}\Bigg\{\left(R'(k_{t1})\mathcal{L}_{\mathrm{NNLL}}(k_{t1}) - \partial_L\mathcal{L}_{\mathrm{NNLL}}(k_{t1})\right)\\
&\times\left(R''(k_{t1})\ln\frac{1}{\zeta_s}+\frac{1}{2}R'''(k_{t1})\ln^2\frac{1}{\zeta_s}\right) - R'(k_{t1})\left(\partial_L\mathcal{L}_{\mathrm{NNLL}}(k_{t1}) - 2\frac{\beta_0}{\pi}\alpha_s^2(k_{t1})\hat{P}^{(0)}\otimes\mathcal{L}_{\mathrm{NLL}}(k_{t1})\ln\frac{1}{\zeta_s}\right)\\
&+\frac{\alpha_s^2(k_{t1})}{\pi^2}\hat{P}^{(0)}\otimes\hat{P}^{(0)}\otimes\mathcal{L}_{\mathrm{NLL}}(k_{t1})\Bigg\}\Big\{\Theta\left(v - V(\{\tilde{p}\},k_1,\dots,k_{n+1},k_s)\right) - \Theta\left(v - V(\{\tilde{p}\},k_1,\dots,k_{n+1})\right)\Big\}\\
&+\frac{1}{2}\int \frac{dk_{t1}}{k_{t1}}\frac{d\phi_1}{2\pi}e^{-R(k_{t1})}\int d\mathcal{Z}[\{R',k_i\}]\int_0^1\frac{d\zeta_{s1}}{\zeta_{s1}}\frac{d\phi_{s1}}{2\pi}\int_0^1\frac{d\zeta_{s2}}{\zeta_{s2}}\frac{d\phi_{s2}}{2\pi}R'(k_{t1})\\
&\times\Bigg\{\mathcal{L}_{\mathrm{NLL}}(k_{t1})\left(R''(k_{t1})\right)^2\ln\frac{1}{\zeta_{s1}}\ln\frac{1}{\zeta_{s2}} - \partial_L\mathcal{L}_{\mathrm{NLL}}(k_{t1})R''(k_{t1})\left(\ln\frac{1}{\zeta_{s1}}+\ln\frac{1}{\zeta_{s2}}\right)\\
&+\frac{\alpha_s^2(k_{t1})}{\pi^2}\hat{P}^{(0)}\otimes\hat{P}^{(0)}\otimes\mathcal{L}_{\mathrm{NLL}}(k_{t1})\Bigg\}\\
&\times\Big\{\Theta\left(v - V(\{\tilde{p}\},k_1,\dots,k_{n+1},k_{s1},k_{s2})\right) - \Theta\left(v - V(\{\tilde{p}\},k_1,\dots,k_{n+1},k_{s1})\right) -\\
&\Theta\left(v - V(\{\tilde{p}\},k_1,\dots,k_{n+1},k_{s2})\right) + \Theta\left(v - V(\{\tilde{p}\},k_1,\dots,k_{n+1})\right)\Big\} + \mathcal{O}\left(\alpha_s^n\ln^{2n-6}\frac{1}{v}\right), \qquad (3.18)
\end{aligned}
$$

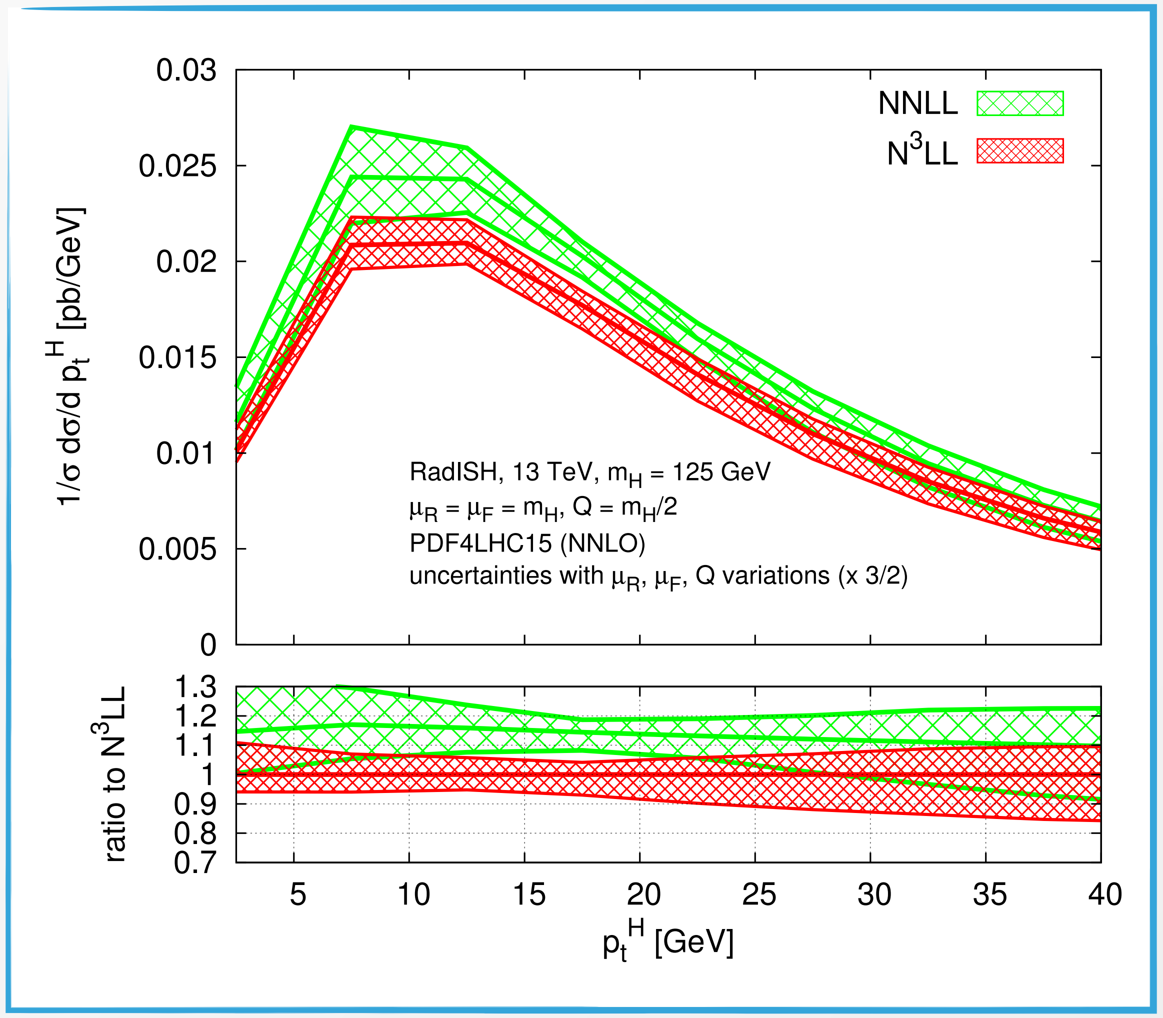
Checks and remarks

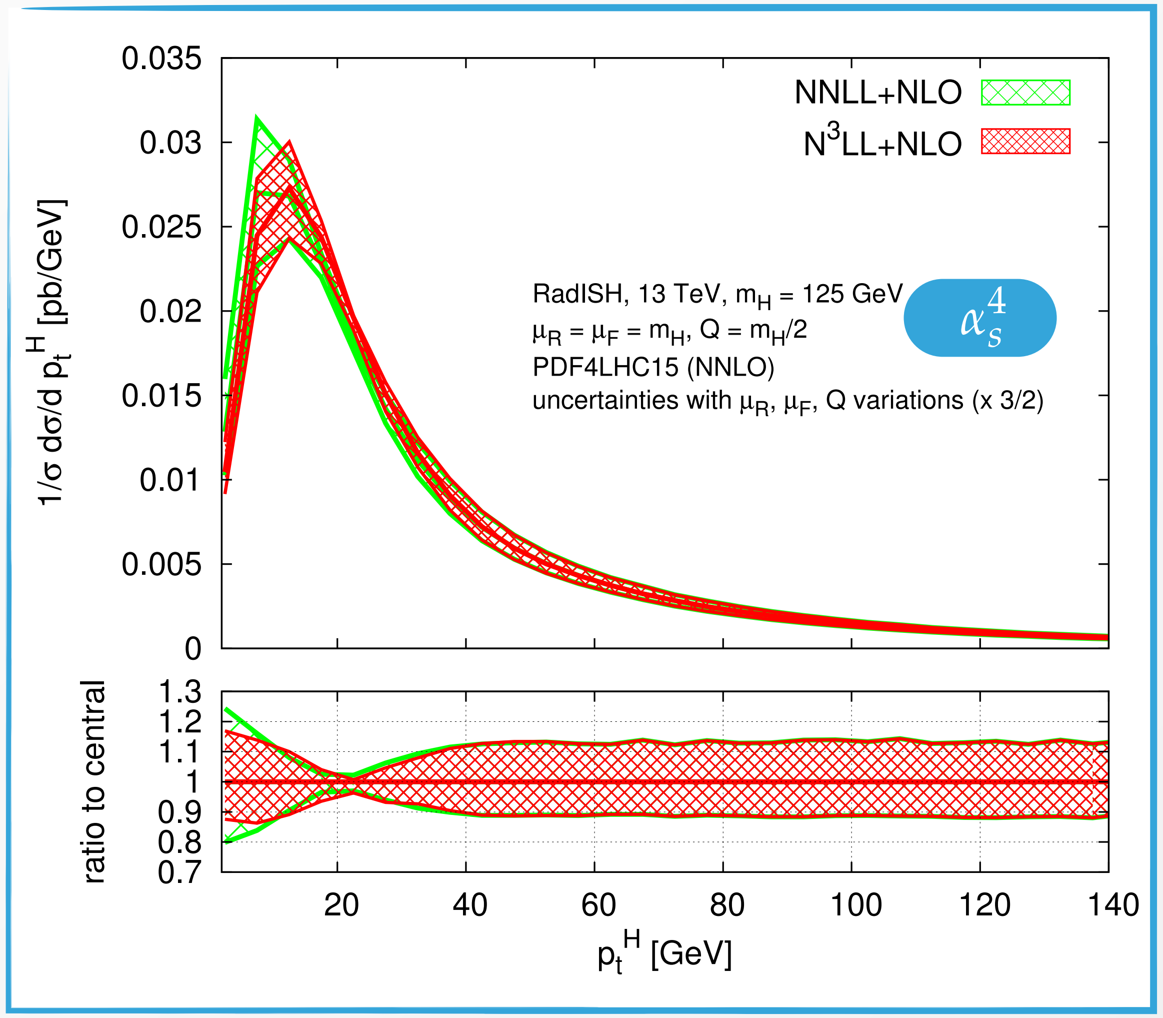
- **b-space** formulation **reproduced analytically** at the resummed level
- **correct scaling** at small p_T computed analytically
- **numerical checks** down to very low p_T against b-space codes (`HqT`, `CuTe`) [Grazzini et al.][Becher et al.]
- check that the FO expansion of the final expression in momentum space up to $O(\alpha^5)$ yields the corresponding expansion in b-space (CSS)
- expansion checked against `MCFM` up to $O(\alpha^4)$ [Campbell et al.]

Matching to fixed order

nb: Cusp anomalous dimension at order α^4 currently unknown set to zero

- Pure N^{3}LL correction amounts to 10-15% (partly induced by the inclusion of the two-loop coefficient functions)
- Residual scale dependence (μ_R,μ_F,Q) ~10%

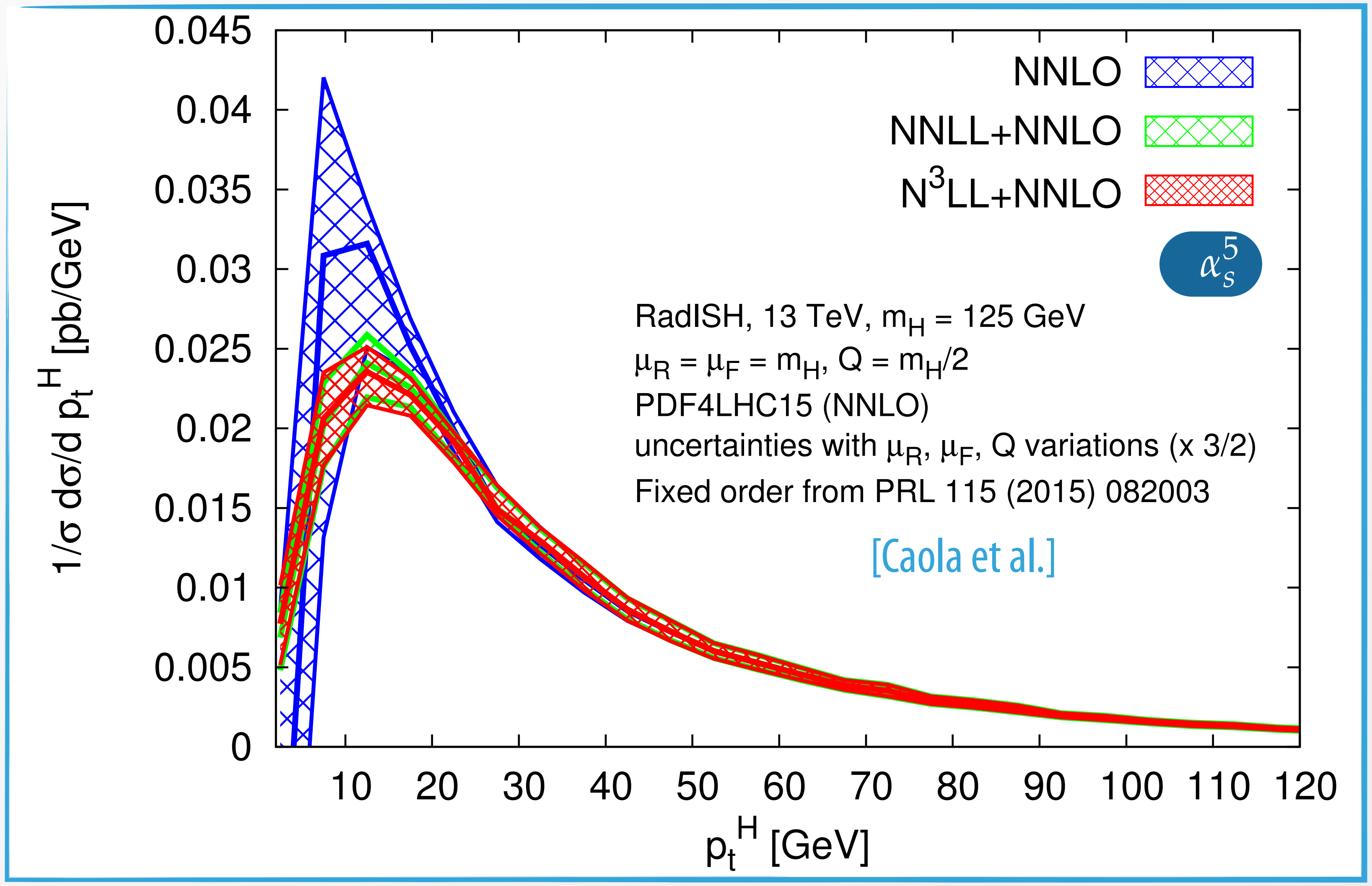




- When matched at NLO, N^{3}LL correction is O(10%) near the peak of the distribution; somewhat larger at small p_T
- Scale ~~uncertainties~~ variations almost halved below 10 GeV, unchanged for larger p_T

Matching to fixed order

- When matched to NNLO, the N^{3}LL correction is a few % at the peak, and *O*(10%) at smaller values of p_T
- Rather moderate reduction of scale dependence at N^{3}LL+NNLO. Need for very stable NNLO distributions below 15 GeV to appreciate reduction. Further runs ongoing
- Mass effects corrections necessary to improve further (see Claudio later)



- Integral of the matched curves yields the N^{3}LO total cross section [Anastasiou et al.]
- Constant terms at N^{3}LO recovered thanks to a **multiplicative scheme matching**

Conclusions Part 1

- New formalism for all-order resummation up to **N^3LL accuracy** for inclusive, transverse observables.
- Method formulated in **momentum space**, does not rely on any specific factorization theorem
- Formally equivalent to the standard b-space formalism
- Method allows for an **efficient implementation in a computer code**. Code `RadISH` can process any colour singlet with arbitrary cuts in the Born phase space. Public release soon.
- Extension to more general transverse observables possible thanks to the universality of the Sudakov radiator

$$V(k) = d_l g_l(\phi) \left(\frac{k_T}{M}\right)^a$$

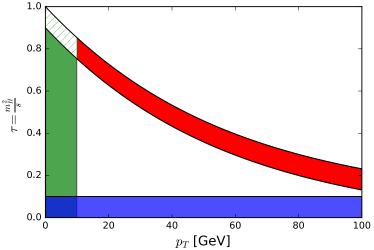
Phenomenological results for the Higgs p_T spectrum:

- N^3LL+NLO correction to the NNLL+NLO spectrum is O(10%) at the peak and below; reduction of scale dependence below the peak.
- N^3LL+NNLO correction to NNLL+NNLO is a few % at the peak and ~10% level below. Moderate reduction of scale dependence, which is now ~10% for the whole spectrum at small p_T

Resummation in the high-p_T region

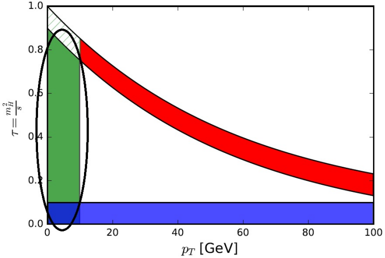
Leaving small-p_{T} region...

Other Resummations are possible...



Leaving small-p_{T} region...

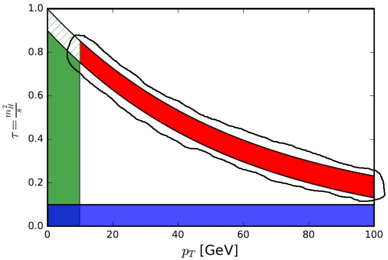
Other Resummations are possible...



- Small-p_{T}: $p_{\mathrm{T}} \lesssim m, \xi_p \ll 0$

Leaving small-p_{T} region...

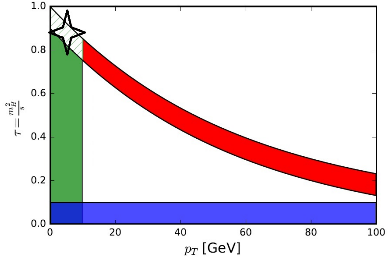
Other Resummations are possible...



- Small-p_{T}: $p_{\mathrm{T}} \lesssim m, \xi_p \ll 0$
- Threshold Limit: $\tau \sim \tau_{\max} = \frac{\sqrt{M^2+p_{\mathrm{T}}^2}-p_{\mathrm{T}}}{M^2}, \tau' \sim 1$

Leaving small-p_{T} region...

Other Resummations are possible...



- Small-p_{T}: $p_{\mathrm{T}} \lesssim m, \xi_p \ll 0$
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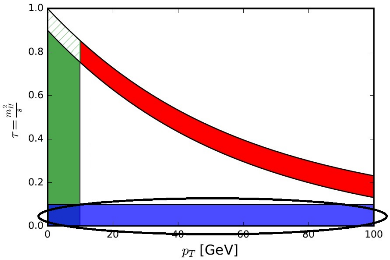

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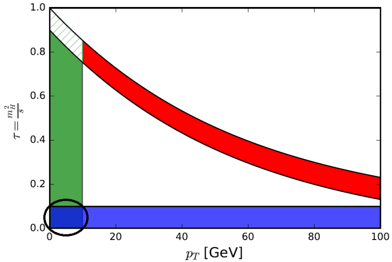

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Combined Resummation

(CM, Forte, Ridolfi, '17)

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$$\frac{d\sigma_{ij}}{dp_{\mathrm{T}}^2}\left(N,p_{\mathrm{T}}^2\right)=\left(1-T\left(N,p_{\mathrm{T}}^2\right)\right)\frac{d\hat{\sigma}_{ij}^{\mathrm{tr}'}}{d\xi_p}\left(N,p_{\mathrm{T}}^2\right)+T\left(N,p_{\mathrm{T}}^2\right)\frac{d\hat{\sigma}_{ij}^{\mathrm{fixed}}}{d\xi_p}\left(N,p_{\mathrm{T}}^2\right)$$

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$$\exp\left[S\left(\chi,N,\alpha_s\left(M^2\right)\right)\right]\sum_{ij}C_i\left(N,\alpha_s\left(\frac{1}{\chi}\right)\right)C_j\left(N,\alpha_s\left(\frac{1}{\chi}\right)\right)f_i\left(N,\frac{1}{\chi}\right)f_j\left(N,\frac{1}{\chi}\right)$$

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$$\chi=\bar{N}^2+\frac{b^2}{b_0^2}\qquad \bar{N}=Ne^{\gamma_E}\qquad b_0=2e^{-\gamma_E}\qquad (1)$$

Matching with small-p_{T} region

$$\frac{d\hat{\sigma}_{ij}^{\text{fixed}}}{d\xi_p}\left(N,\xi_p,\alpha_s\left(Q^2\right),Q^2\right)=\sigma_0\,C_{0,ij}\left(N,\xi_p\right)g_{0,ij}\left(\xi_p,\alpha_s\right)\exp\left[G\left(N,\alpha_s\right)\right]\exp\left[S\left(N,\xi_p,\alpha_s\right)\right]$$

Problem!

At small-p_{T}:

$$\frac{d\hat{\sigma}_{ij}^{\text{fixed}}}{d\xi_p}\sim\alpha_s^n\frac{\ln^n\xi_p}{\xi_p}\ln^{n-1}N \tag{2}$$

while fixed order calculations and small-p_{T} resummation predict

$$\frac{d\hat{\sigma}_{ij}}{d\xi_p}\sim\alpha_s^n\frac{\ln^{n-1}\xi_p}{\xi_p}\ln N \tag{3}$$

Soft behaviour completely wrong at small-p_{T} since new soft configurations arise, previously suppressed by the finite value of p_{T}.

Matching with small-p_{T} region

Problem!

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Another Problem!

Fixed-order calculations and threshold Resummation at fixed p_{T} at large N:

$$\frac{d\hat{\sigma}_{ij}^{\mathrm{fixed}}}{d\xi_p} \sim \alpha_s^n \frac{1}{\sqrt{N}} \ln^{2n-1} N \tag{2}$$

while CSS small-p_{T} resummation at large N scales as

$$\frac{d\hat{\sigma}_{ij}^{\mathrm{small-p_T res}}}{d\xi_p} \sim \alpha_s^n \ln N \tag{3}$$

At large-p_{T}, small-p_{T} resummation shows a not-physical logarithmic behaviour at large N

Matching with small-p_{T} region

How to solve these problems? Phase space Analysis (CM, Forte, Ridolfi, '17)
At small-p_{T}, phase-space for n emissions factorizes in Mellin-Fourier space:

$$d\Phi_{n+1}\left(p_1, p_2; p, k_1, \ldots, k_n\right) = M^{2n} \frac{8\pi^3}{\left[4(2\pi)^2\right]^{n+1}} d\xi_p \int db^2\, J_0\left(bp_{\mathrm{T}}\right)$$
$$J_0\left(bk_{\mathrm{T}_1}\right) \frac{d\xi_1 dz_1}{\sqrt{\left(1-z_1\right)^2 - 4\xi_1}} \ldots J_0\left(bk_{\mathrm{T}_\mathrm{n}}\right) \frac{d\xi_n dz_n}{\sqrt{\left(1-z_n\right)^2 - 4\xi_n}}$$
$$\delta\left(\hat{\tau} - z_1 \ldots z_n\right) + \mathcal{O}\left(\frac{1}{b}\right). \tag{4}$$

Now standard p_{T} resummation considers $\xi_i = \frac{p_{\mathrm{T,i}}^2}{M^2} \ll \left(1-z_i\right)^2$ and rewrites the square-root as

$$\frac{1}{\sqrt{\left(1-z\right)^2 - 4\xi}} \to \left(\frac{1}{1-z}\right)_+ - \frac{1}{2}\delta\left(1-z\right)\ln\xi \tag{5}$$

By taking this limit:

- We destroy the large-N behaviour at fixed-p_{T}

$$\mathcal{M}\left[\frac{1}{\sqrt{(1-z)^2-4\xi}}\right] \sim \frac{1}{\sqrt{N}} \qquad \mathcal{M}\left[\left(\frac{1}{1-z}\right)_+\right] \sim \ln N \tag{6}$$

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- This approximation ruins when $(1-z)^2 \sim \xi$, which in Mellin-Fourier space means

$$\mathcal{F}\mathcal{M}\left[\frac{1}{\sqrt{(1-z)^2-4\xi}}\right] = \frac{2}{b^2}\left(1-\frac{4N^2}{b^2}+\frac{16N^4}{b^4}+\dots\right), \qquad (7)$$

we are missing terms suppressed by powers of b but enhanced with the same powers of N.

By taking this limit:

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- Integral over ξ can not be right since

$$\int_0^{\frac{(1-z)^2}{4}}d\xi\,\frac{1}{\sqrt{(1-z)^2-4\xi}}=\frac{(1-z)}{4}\left(1+\frac{1}{4}+\frac{1}{8}+\dots\right) \tag{8}$$

after integration all the terms in the expansion are of the same order

JOINT RESUMMATION

Combined Resummation

(CM, Forte, Ridolfi, '17)

Combined Resummation

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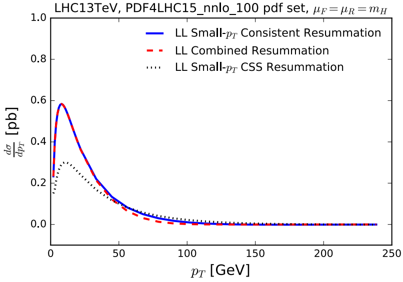
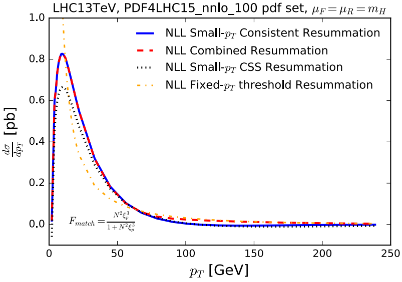
Preliminary Results for Higgs p_T distribution

Resummed Component... no matching with fixed order

LL



NLL



PRELIMINARY (Forte, Muselli, Ridolfi in preparation)

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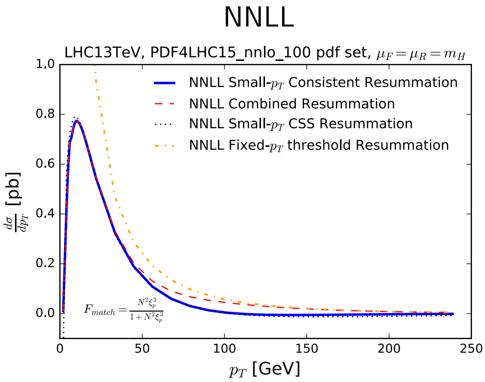

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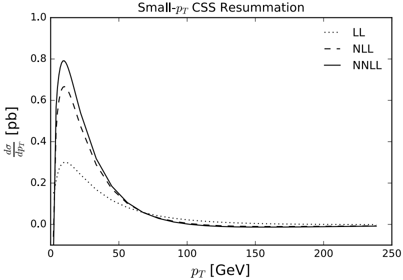
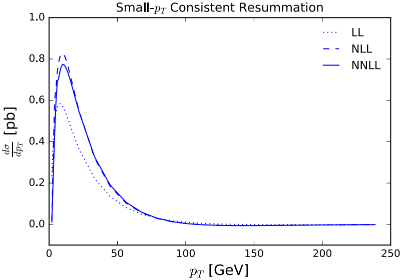


Consistent



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- Moreover, integral of combined resummation coincides with threshold resummed inclusive cross section at the same logarithmic accuracy, as in Joint Resummation.
- First preliminary analysis on Higgs boson distribution shows a small impact at small-p_{T} at NNLL, but an improvement in the convergence of the resummed series.

High Energy Resummation

High Energy Resummation $\ln \frac{M^2}{s}$ was developed recently for p_{T} distributions

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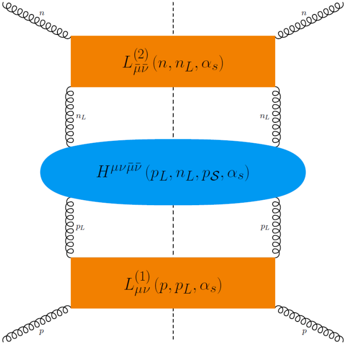

k_{T} Factorization

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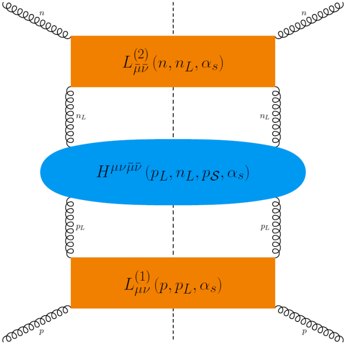

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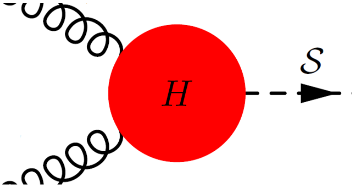

Off-shell $p_{\rm T}$ distribution

LL Resummation

High Energy Resummation

As regards the Higgs p_{T} distribution this technique was applied:

- in EFT framework.(Forte, CM, '15)

$$h_{p_{\mathrm{T}}} = R(M_1)\,R(M_2)\,\sigma_{\mathrm{LO}}\,\frac{\xi_p^{M_1+M_2-1}}{(1+\xi_p)^N}\left[\frac{\Gamma(1+M_1)\,\Gamma(1+M_2)\,\Gamma(2-M_1-M_2)}{\Gamma(2-M_1)\,\Gamma(2-M_2)\,\Gamma(M_1+M_2)}\left(1+\frac{2M_1M_2}{1-M_1-M_2}\right)\right] \tag{10}$$

- with top e bottom quark contribution. (Caola, Forte, Marzani, CM, Vita, '16)

$$h_{p_{\mathrm{T}}} = R(M_1)\,R(M_2)\,\sigma_{\mathrm{LO}}(y_i)\,\frac{\xi_p^{M_1+M_2-1}}{(1+\xi_p)^N}\left[c_0(\xi_p,y_i)\,(M_1+M_2)+\sum_{j\geq k>0}c_{j,k}(\xi_p,y_i)\left(M_1^j\,M_2^k+M_1^k\,M_2^j\right)\right] \tag{11}$$

This analysis brought some remarks about the all-order structure of quark mass contributions in the high-p_{T} region for Higgs.
Quark Mass Effect will be the main subject of my next slides.

High Energy Resummation at Small-p_{T}

Double Resummation small-p_{T} / high energy was also recently studied (Marzani, '16)

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$$\frac{d\hat{\sigma}_{ij}}{dp_{\mathrm{T}}^2} = \sigma_0 \int db \, \frac{b}{2} J_0\left(bp_{\mathrm{T}}\right) H\left[C\left(N\right) C\left(N\right) + G\left(N\right) G\left(N\right)\right] S\left(b\right) \Gamma_i\left(N, \frac{1}{b}\right) \Gamma_j\left(N, \frac{1}{b}\right) \tag{12}$$

$$\frac{d\hat{\sigma}_{gg}^{\mathrm{h.e}}}{dp_{\mathrm{T}}^2} = \sigma_0 \int db \, \frac{b}{2} J_0\left(bp_{\mathrm{T}}\right) \left[\frac{\Gamma\left(1+\gamma\left(\alpha_s, N\right)\right)^2}{\Gamma\left(1-\gamma\left(\alpha_s, N\right)\right)} + \gamma\left(\alpha_s, N\right)^2 \frac{\Gamma\left(1+\gamma\left(\alpha_s, N\right)\right)^2}{\Gamma\left(2-\gamma\left(\alpha_s, N\right)\right)}\right] e^{-2\gamma\left(\alpha_s, N\right) \ln \frac{b^2}{b_0^2}} \tag{13}$$

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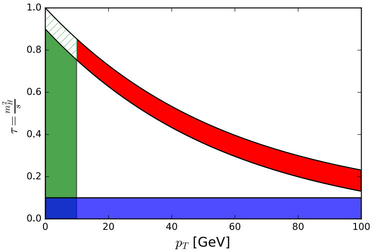
Small-x resummation is then simply achieved by:

- performing the evolution of PDFs with an anomalous dimension resummed at small-x.
- resumming the hard function

$$C\left(N\right) = C^{\mathrm{small-p_T}}\left(N\right) + C^{\mathrm{he}}\left(N\right)$$

$$G\left(N\right) = G^{\mathrm{small-p_T}}\left(N\right) + G^{\mathrm{he}}\left(N\right)$$

Closing about Resummations



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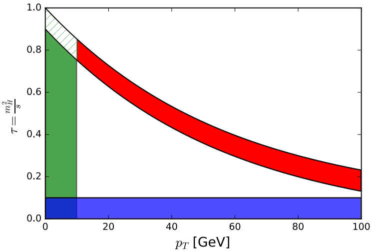

Triple Joint Resummation is right behind the corner!

Mass effects

Mass quark Effects

Great improvements in the last two years.
Contributions arose with several different approaches:

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- High-energy Resummation (Caola, Forte, Marzani, CM, Vita, '16)
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Contributions arose with several different approaches:

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I will try to summarize the state of the art... difficult task!

HE Resummation: all-order structure

Using High-Energy Resummation we came to two important all-order conclusions (Caola, Forte, Marzani, CM, Vita, '16)

1. At large-p_{T}, leading terms at high energy owns the same behaviour at any order both in EFT and in full SM, once we rescale the series for the LO contribution.

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These conclusions are valid at all orders in α_s in the high energy regime but, however, accuracy is not higher enough to approximate properly the full NLO.

We need to combine these results with also other resummations (threshold, small-$p_{\rm T}$...) to become precise $\Rightarrow$ LH19

Pro

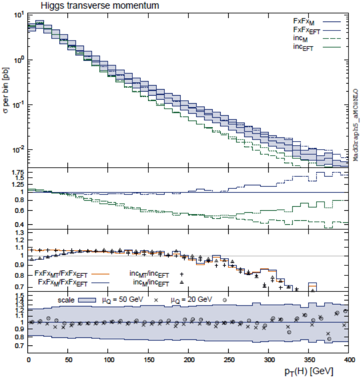
All-order analysis

Contra

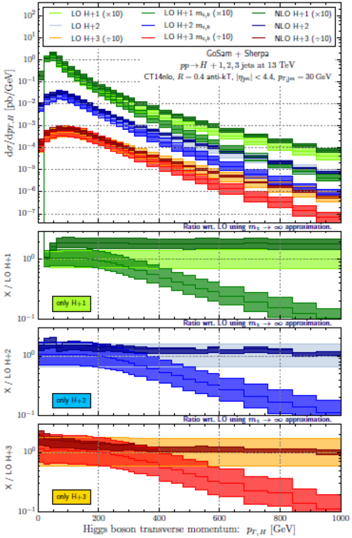
Uncertainty still quite large, up to now

Matched Parton Showers: top contributions

MG5 FxFx



GoSam+Sherpa



- Real Massive Diagrams are computed exactly.
- Virtual massive two-loops contribution is absent. It could be negligible for top, probably not for bottom.
- Same p_T dependence raising the number of jets, as highlighted by high energy resummation.

Series Expansion in $\frac{1}{m_{\mathrm{top}}}$

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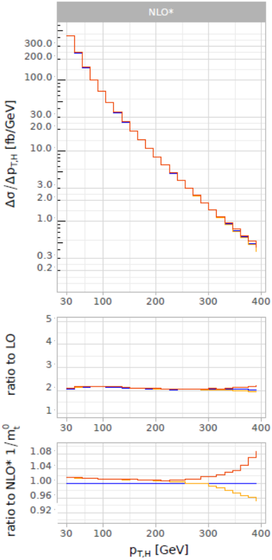

- The best approximation of NLO is obtained by merging the exact full SM real emission with virtual two-loops correction obtained as expansion in power of $\frac{1}{m_{top}}$.
- In the two loops virtual contribution the dependence from the order of expansion is around 20% already at $200GeV$.
- Moreover, this approximation totally fails approaching the top pair production threshold $p_{\mathrm{T}} \sim 2m_{\mathrm{top}}$
- However, final NLO* approximation seems to be quite stable in the region around $p_{\mathrm{T}} \sim m_{\mathrm{top}}$

Personal Considerations

- Estimates about m_{top} effects for moderate large-p_{T} produced by:
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- I think that LH17 could be the right place to discuss about possible benchmarks among these approaches to provide a robust uncertainty at NLO for top contributions (argument for discussion).
- I also think that these approaches are now not the best options in considering the impact of bottom contributions. Fortunately new results are also coming on this side.

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- On my opinion, however, exponentiation in general of these Logs remains an open question and a possible argument of discussion

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- I really think that LH17 can be the right place to unify all these different analysis to propose a reliable and robust uncertainty for top and bottom contributions on the p_{T} distribution of Higgs at NLO.
- Many improvements in the last two years have been made also in the context of resummations theories. I am really confident that by combining all the information coming from these different regimes, we are soon be able to approximate at the level of $5-10\%$ higher order contributions to Higgs differential distributions.

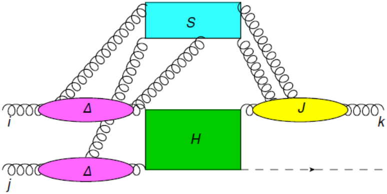
Backup

Threshold Resummation at fixed-$p_{\rm T}$: $\frac{d\hat{\sigma}_{ij}^{\rm fixed}}{d\xi_p}$

(De Florian, Kulesza, Vogelsang, '05)
(CM, Forte, Ridolfi, '17)

$$\frac{d\hat{\sigma}_{ij}^{\rm fixed}}{d\xi_p}\left(N,\xi_p,\alpha_s\left(Q^2\right),Q^2\right)=\sigma_0\,C_{0,ij}\left(N,\xi_p\right)g_{0,ij}\left(\xi_p,\alpha_s\right)\exp\left[G\left(N,\alpha_s\right)\right]\exp\left[S\left(N,\xi_p,\alpha_s\right)\right]$$

$$G\left(N,\alpha_s\right)=\Delta_i\left(N,\alpha_s\right)+\Delta_j\left(N,\alpha_s\right)+J_k\left(N,\alpha_s\right) \qquad \xi_p=\frac{p_{\rm T}^2}{m_{\rm H}^2}$$



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$$\Delta_i\left(N,\alpha_s\right)=\int_0^1 dz\,\frac{z^{N-1}-1}{1-z}\int_{Q^2}^{Q^2(1-z)^2}\frac{dq^2}{q^2}A_i^{\text{th}}\left(\alpha_s\left(q^2\right)\right)\tag{14}$$

$$J_k\left(N,\alpha_s\right)=\int_0^1 dz\,\frac{z^{N-1}-1}{1-z}\int_{Q^2(1-z)^2}^{Q^2(1-z)}\frac{dq^2}{q^2}A_k^{\text{th}}\left(\alpha_s\left(q^2\right)\right)+B_k^{\text{th}}\left(\alpha_s\left(Q^2\left(1-z\right)\right)\right)\tag{15}$$

$$S\left(N,\xi_p\right)=-\int_0^1 dz\,\frac{z^{N-1}-1}{1-z}A_k^{\text{th}}\left(\alpha_s\left(Q^2\left(1-z\right)^2\right)\right)\ln\frac{\left(\sqrt{1+\xi_p}+\sqrt{\xi_p}\right)^2}{\xi_p}\tag{16}$$

with A^{th} the cusp anomalous dimension.