Loop-Tree Duality as a new regularization scheme

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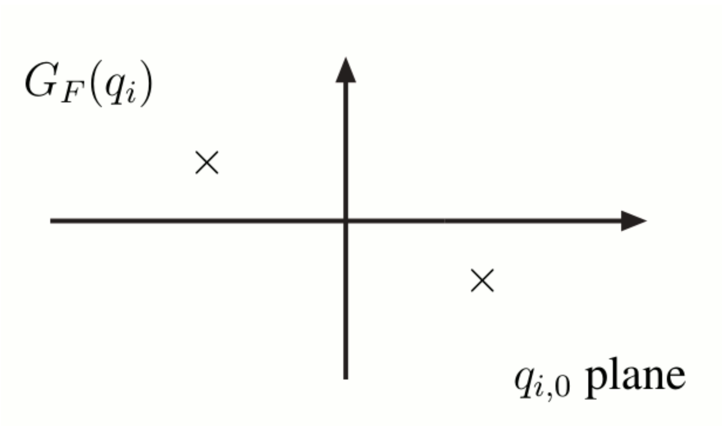
Comparison with DREG

DREG	LTD / FDU
▪ Modify the dimensions of the space-time to **d = 4-2e**	▪ Computations without altering the **d=4 space-time** dimensions[1]
▪ Singularities manifest **after** integration as **1/e poles:** ▪ **IR** cancelled through suitable **subtraction terms,** which need to be integrated over the unresolved phase-space ▪ **UV** renormalized	▪ Singularities killed **before** integration: ▪ **Unsubtracted** summation over degenerate IR states at integrand level through a suitable **momentum mapping** ▪ **UV** through local counter-terms
▪ Virtual and real contributions are considered **separately:** phase-space with **different number of final-state particles**	▪ Virtual and real contributions are considered **simultaneously:** more efficient Monte Carlo implementation and fully differential

[1] Gnendiger et al., *To d, or not to d: Recent developments and comparisons of regularization schemes*, arXiv:1705.01827

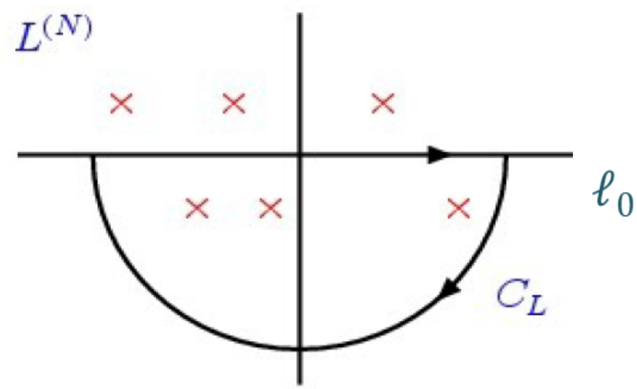
The loop-tree duality theorem

[Catani et al. 2008]



Cauchy residue theorem

in the loop energy complex plane



Feynman Propagator +i0:
positive frequencies are propagated forward in time, and negative backward.

selects residues with definite **positive energy and negative imaginary part** (indeed in any coordinate system)

$$G_F(q_i) = \frac{1}{q_i^2 - m_i^2 + i0} \,, \qquad q_i = \ell + \sum_{k=1}^{i} p_k$$

The loop-tree duality theorem

[Catani et al. 2008]

One-loop integrals (or scattering amplitudes in any relativistic, local and unitary QFT) represented as a linear combination of *N* **single-cut phase-space** integrals

$$\int_\ell \prod G_F(q_i) = -\sum \int_\ell \tilde{\delta}(q_i) \prod_{j \neq i} G_D(q_i; q_j)$$

- $\tilde{\delta}(q_i) = i\,2\pi\,\theta(q_{i,0})\,\delta(q_i^2 - m_i^2)$ sets internal line on-shell, positive energy mode
- $G_D(q_i; q_j) = \dfrac{1}{q_j^2 - m_j^2 - i0\,\eta\,k_{ji}}$ **dual propagator,** $k_{ji} = q_j - q_i$

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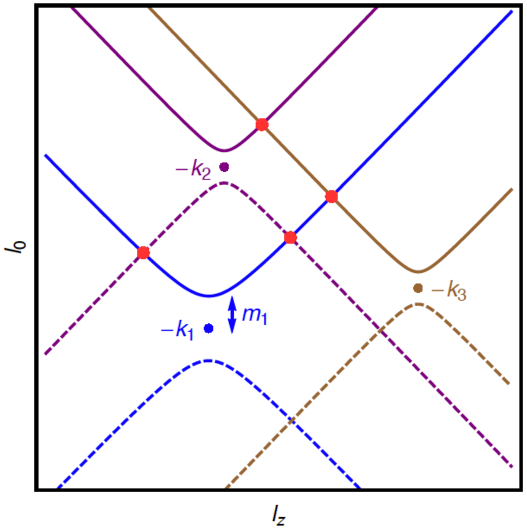
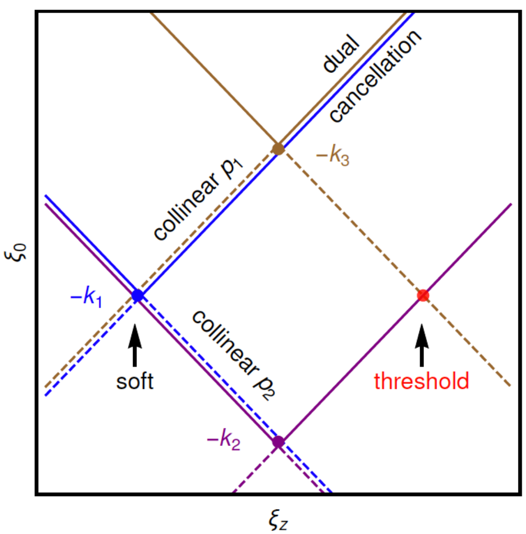
- Lorentz-covariant dual prescription with η a **future-like** vector; from now $\eta^\mu = (1, \mathbf{0})$ only the **sign** matters

Singularities of the loop integrand

Buchta et al, **arXiv:1405.7850, JHEP11(2014)014**

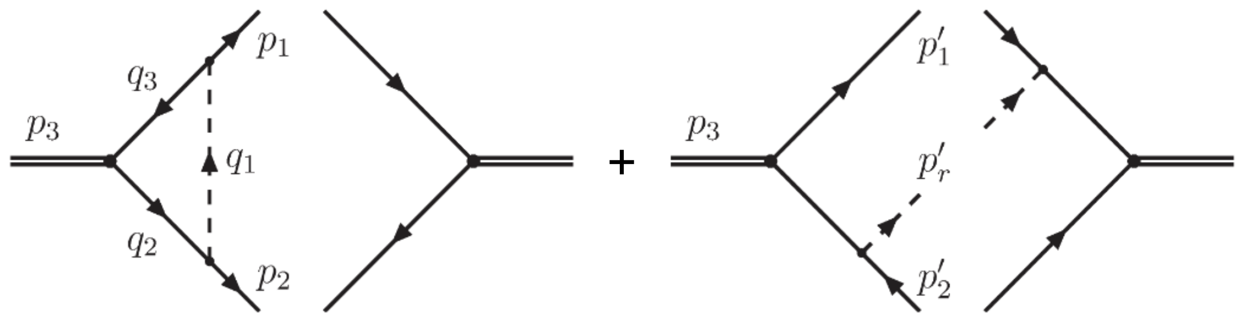




- **LTD:** equivalent to integrating along the **forward** on-shell hyperboloids / light-cones (positive energy modes)
- The dual loop integrand becomes singular when subsets (>=2) of internal propagators go on-shell while integrating
- **Cancellations** of singularities among dual amplitudes at forward-forward intersections: dual +i0 prescription changes sign, proof of consistency
- Only backward (negative energy) with forward IR and threshold singularities remain: **time-like** separated propagators with lower energy causally connected

IR and threshold singularities are restricted to a **compact region** of the loop three-momentum

IR: adding virtual and real contributions



- Singularities appearing in the virtual and real contributions have **different signs**
- Within LTD framework, cancellations must be performed **locally**
- We need to generate a **1→3 kinematics** starting from a **1→2 configuration plus the loop three-momentum**

IR: momentum mapping

Rodrigo et al, **arXiv:1604.06699, JHEP08(2016)160**

Defining the mappings requires two steps:

- Splitting the real phase space into two regions, i.e. $y'_{1r}<y'_{2r}$ and $y'_{2r}<y'_{1r}$, to separate the possible collinear singularities (there cannot be more than one in a given region of the phase-space)

- Implementing an optimized mapping in each region, to allow a fully local cancellation of IR singularities with those present in the dual contributions

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REGION 1:

$$p'^{\mu}_r = q^{\mu}_1 \,, \qquad p'^{\mu}_1 = p^{\mu}_1 - q^{\mu}_1 + \alpha_1 \, p^{\mu}_2 \,,$$
$$p'^{\mu}_2 = (1-\alpha_1)\, p^{\mu}_2 \,, \qquad \alpha_1 = \frac{q_3^2}{2 q_3 \cdot p_2} \,,$$

$\Rightarrow$

$$y'_{1r} = \frac{v_1 \, \xi_{1,0}}{1-(1-v_1)\,\xi_{1,0}} \qquad y'_{12} = 1 - \xi_{1,0}$$
$$y'_{2r} = \frac{(1-v_1)(1-\xi_{1,0})\,\xi_{1,0}}{1-(1-v_1)\,\xi_{1,0}}$$

REGION 2:

$$p'^{\mu}_2 = q^{\mu}_2 \,, \qquad p'^{\mu}_r = p^{\mu}_2 - q^{\mu}_2 + \alpha_2 \, p^{\mu}_1 \,,$$
$$p'^{\mu}_1 = (1-\alpha_2)\, p^{\mu}_1 \,, \qquad \alpha_2 = \frac{q_1^2}{2 q_1 \cdot p_1} \,,$$

$\Rightarrow$

$$y'_{1r} = 1 - \xi_{2,0} \qquad y'_{2r} = \frac{(1-v_2)\,\xi_{2,0}}{1-v_2\,\xi_{2,0}}$$
$$y'_{12} = \frac{v_2\,(1-\xi_{2,0})\,\xi_{2,0}}{1-v_2\,\xi_{2,0}}$$

IR: mapping for massive particles

- Rewrite **emitter** and **spectator** in terms of two massless momenta

$$p_i^\mu = \beta_+ \hat{p}_i^\mu + \beta_- \hat{p}_j^\mu$$
$$p_j^\mu = (1-\beta_+)\,\hat{p}_i^\mu + (1-\beta_-)\,\hat{p}_j^\mu \qquad \hat{p}_i^\mu + \hat{p}_j^\mu = p_i^\mu + p_j^\mu$$

- Mapping and phase-space partition formally equal to the massless case: determine **mapping parameters from on-shell conditions**

$$p_r'^\mu = q_i^\mu \ ,$$
$$p_i'^\mu = (1-\alpha_i)\,\hat{p}_i^\mu + (1-\gamma_i)\,\hat{p}_j^\mu - q_i^\mu \ ,$$
$$p_j'^\mu = \alpha_i\,\hat{p}_i^\mu + \gamma_i\,\hat{p}_j^\mu \ , \qquad p_k'^\mu = p_k^\mu \ , \quad k \neq i,j$$

- **Quasi-collinear configurations** are conveniently mapped such that the massless limit is smooth

UV renormalization: local subtraction

- Expand propagators around a UV propagator [Weinzierl et al. 2010]

$$G_F(q_i) = \frac{1}{q_{\rm UV}^2 - \mu_{\rm UV}^2 + i0} + \ldots \qquad q_{\rm UV} = \ell + k_{\rm UV}$$

- and adjust **subleading** terms to subtract only the pole ($\overline{\rm MS}$ **scheme**), or to define any other renormalisation scheme. For the scalar two point function

$$I_{\rm UV}^{\rm cnt} = \int_\ell \frac{1}{(q_{\rm UV}^2 - \mu_{\rm UV}^2 + i0)^2}$$

- Dual representation needs to deal with **multiple poles** [Bierenbaum et al.]

$$I_{\rm UV}^{\rm cnt} = \int_\ell \frac{\tilde\delta(q_{\rm UV})}{2\left(q_{\rm UV,0}^{(+)}\right)^2}$$

$$q_{\rm UV,0}^{(+)} = \sqrt{\mathbf{q}_{\rm UV}^2 + \mu_{\rm UV}^2 - i0}$$

Hernández-Pinto, Sborlini, GR, arXiv:1506.04617

Self-energy corrections

- **Wave function** corrections usually **ignored for massless partons**, but they feature non-trivial IR/UV behavior, required to disentangle both regions, indeed necessary to map the squares of the real amplitudes in the IR

- **Integrand-level** expression for the wave-function and mass renormalization (for quarks)**:**

$$\Delta Z_2(p_1) = -g_{\rm S}^2 C_F \int_\ell G_F(q_1)\, G_F(q_3) \left((d-2)\frac{q_1\cdot p_2}{p_1\cdot p_2} + 4M^2\left(1-\frac{q_1\cdot p_2}{p_1\cdot p_2}\right) G_F(q_3)\right)$$

$$\Delta Z_M^{\rm OS}(p_1) = -g_{\rm S}^2 C_F \int_\ell G_F(q_1)\, G_F(q_3) \left((d-2)\frac{q_1\cdot p_2}{p_1\cdot p_2} + 2\right)$$

For a similar discussion, see [Weinzierl et al. 2016]

⇨ Smooth massless limit

- And locally **subtract the UV**

$$\Delta Z_2^{\rm UV}(p_1) = -(d-2) g_{\rm S}^2 C_F \int_\ell (G_F(q_{\rm UV}))^2 \left(1+\frac{q_{\rm UV}\cdot p_2}{p_1\cdot p_2}\right)$$
$$\times\,(1-G_F(q_{\rm UV})(2q_{\rm UV}\cdot p_1+\mu_{\rm UV}^2))$$

LTD unsubtraction: multi-leg

Sborlini, FDM, Hernández-Pinto, Rodrigo, **arXiv:1604.06699, JHEP08(2016)160**

- The **dual representation** of the renormalized loop cross-section: one single integral in the loop three-momentum

$$\int_m d\sigma_{\mathrm{V}}^{(1,R)} = \sum_{i=1}^{N} \int_m \int_\ell 2\,\mathrm{Re}\,\langle \mathcal{M}_N^{(0)} | \mathcal{M}_N^{(1,R)}(\tilde{\delta}(q_i))\rangle\, \mathcal{O}_N(\{p_j\})$$

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- A **partition** of the real phase-space

$$\sum \mathcal{R}_i(q_i, p_i) = \sum \prod_{jk \neq ir} \theta(y'_{jk} - y'_{ir}) = 1$$

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- A **partition** of the real phase-space

$$\sum \mathcal{R}_i(q_i,p_i) = \sum \prod_{jk\neq ir} \theta(y'_{jk} - y'_{ir}) = 1$$

- The real contribution **mapped** to the Born kinematics + loop three-momentum

$$\int_{m+1} d\sigma_{\rm R}^{(1)} = \sum_{i=1}^{N} \int_{m+1} |\mathcal{M}_{N+1}^{(0)}(q_i,p_i)|^2\, \mathcal{R}_i(q_i,p_i)\, \mathcal{O}_{N+1}(\{p'_j\})$$

- with

$$p'^{\mu}_r = q^\mu_i \ ,$$

$$p'^{\mu}_i = p^\mu_i - q^\mu_i + \alpha_i\, p^\mu_j \ , \qquad \alpha_i = \frac{(q_i-p_i)^2}{2p_j\cdot(q_i-p_i)} \ ,$$

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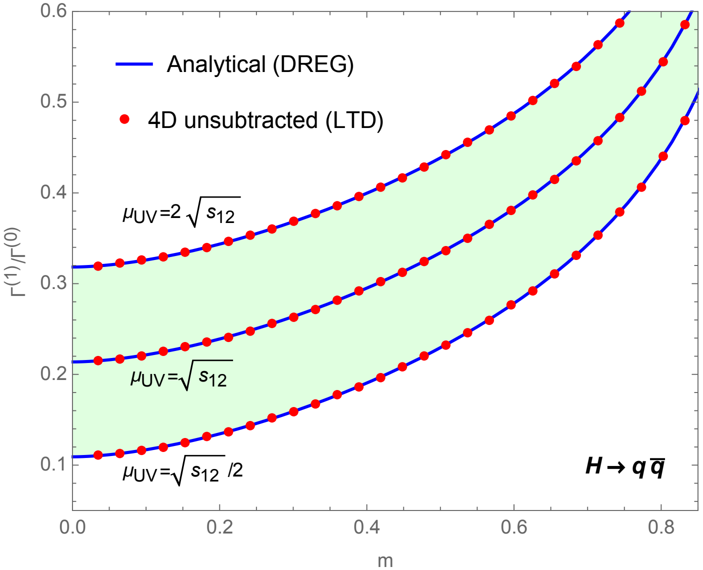
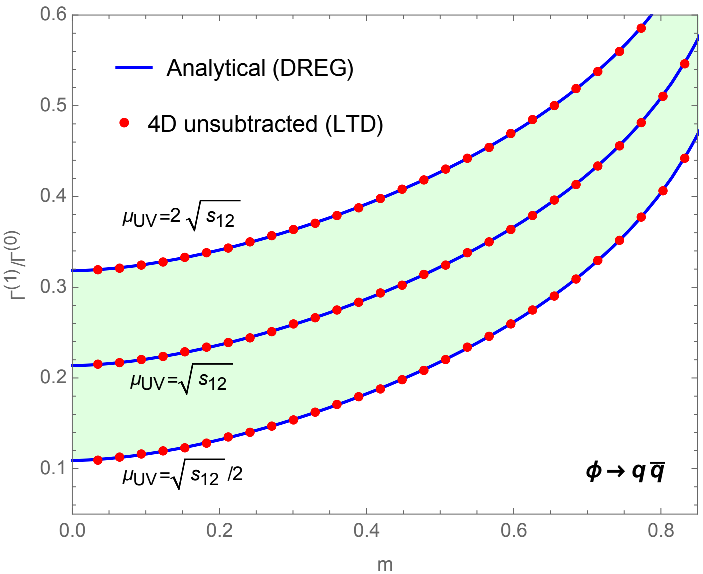
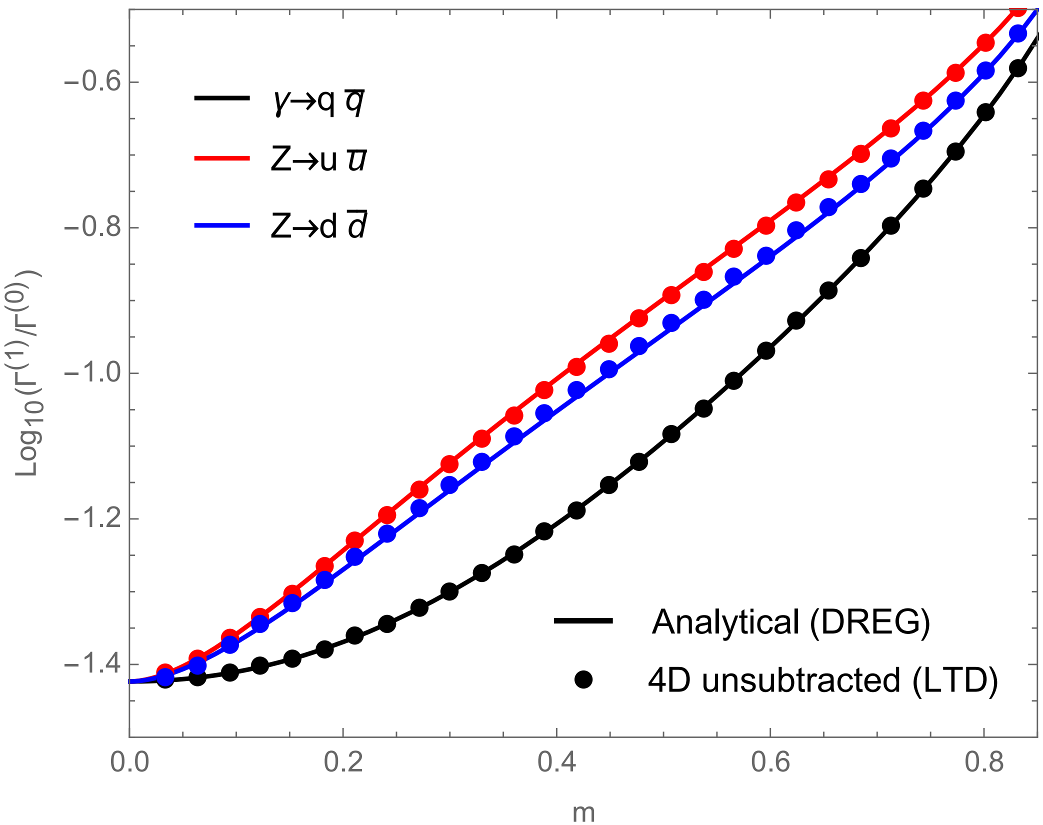
Benchmark application: $A^* \to q\bar{q}(g)$

Sborlini, FDM, Rodrigo, **arXiv:1608.01584, JHEP10(2016)162**







- Excellent agreement with analytic DREG
- Efficient numerical implementation
- Smooth massless limit

Direct asymptotic expansion

FDM, Rodrigo, Sborlini, **arXiv:1702.07581**

- Integration domain is an **Euclidean** space (loop three-momentum)
- **Asymptotic expansions** (heavy or light internal mass) more direct at integrand level than **Minkowsky**

$$\frac{\delta(\ell^2 - M^2)}{s_{12} + 2\ell \cdot p_{12}} = \frac{\delta(\ell^2 - M^2)}{2\ell \cdot p_{12}} \sum_{n=0} \left(\frac{-s_{12}}{2\ell \cdot p_{12}} \right)^n$$

- Each term of the integrand expansion less UV singular than the previous one
- Circumvent **expansion by regions** [Smirnov, Beneke]
- Work still in progress (already tested for gg->H and H->γγ)

Conclusions

- **New algorithm/regularization scheme** for higher-orders in perturbative QFT based on LTD: summation over degenerate soft, final-state collinear singularities and quasi-collinear configurations achieved through a mapping of momenta between real and virtual kinematics.
- **IR unsubtracted and four-dimensional:** fully local cancellation of IR and UV singularities.
- **Smooth massless limit** due to proper treatment of quasi-collinear configurations
- **Threshold singularities** through contour deformation in the loop three-momentum.
- **Simultaneous generation** of real and virtual corrections **advantageous,** particularly for multi-leg processes.
- Direct **asymptotic expansions**.

Outlook: automation and fully differential multi-leg at NNLO (and beyond)

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