Physics of Extra Dimensions at TeV Colliders

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Don Quixote

Don Quixote: "Confess that in all the world there is no maiden fairer than the Empress of La Mancha, the peerless Dulcinea del Toboso."
Trader: "Sir Knight, we do not know who this good lady is that you speak of; show her to us, for, if she be of such beauty as you suggest, with all our hearts and without any pressure we will confess the truth that is on your part required of us."
Don Quixote: "If I were to show her to you, what merit would you have in confessing a truth so manifest? The essential point is that without seeing her you must believe, confess, affirm, swear, and defend it...

Introduction

- General Relativity $\Longrightarrow$ gravity from the geometry of space-time.
- Kaluza-Klein theories, $\Longrightarrow$ gravity and electromagnetism from the geometry of an underlying higher-dimensional space-time.
- Traditional wisdom: The extra dimensions have to be compactified to small and unobservable sizes.
- Post-modern thought: Developments in string theory have led to novel ideas of compactification. So large extra dimensions or small but physically interesting extra dimensions have become possibilities.
- These new ideas lead to models that have a host of implications for high-energy physics, astrophysics and cosmology.
- In this talk, I will first discuss a few of these models and their implications mainly for high-energy physics.

The Problems: Hierarchy and the Cosmological Constant

- Hierarchy problem: Fundamental scale of gravity is the Planck scale $M_P \sim 10^{18}$ GeV. The electroweak scale is $v = 246$ GeV. Problem: To stabilise this very large hierarchy between these two scales with respect to quantum corrections.
- Cosmological constant problem: Why is the measured vacuum energy density so small compared to the electroweak scale?
- The solutions to the hierarchy and the cosmological constant problems become the major goals of theories purporting to go beyond the SM.

Brief Look at Kaluza-Klein Reduction

- Consider a scalar field in 5 dimensions, $\phi(x, y)$. The fifth dimension, y, is compactified to a circle with radius R.
- The 5-dimensional field can be expanded in a Fourier series as

$$\phi(x, y) = \sum_{n=-\infty}^{\infty} \phi_n(x) \exp(iny/R), \tag{1}$$

  where n is an integer and $\phi_n(x)$ are 4-dimensional fields.
- Substituting this in the 5-dimensional Klein-Gordon equation which $\phi(x, y)$ satisfies $\Longrightarrow$ infinite number of equations in 4 dimensions, one for each $\phi_n(x)$ and with a mass $|n|/R$ for every mode n. These modes are called $pyrgons$.
- The definition of spin in D dimensions depends on the D-dimensional Lorentz symmetry. The light-cone symmetry that leaves the motion of a massless particle unchanged in D dimensions is $SO(D-2)$ and the D-dimensional helicity corresponds to the representations of $SO(D-2)$. A given Kaluza-Klein level in $D = 4 + m$ dimensions has one spin-2 state, $(m-1)$ spin-1 states and $m(m-1)/2$ spin-0 states.

Strings and Branes

- It has become possible to study string theories at strong coupling as duals of other theories (at weak coupling). In the strong coupling limit, other than open and/or closed strings solitonic modes appear, which go under the name of Dp-branes.
- Dp-brane $\Longrightarrow$ a $p+1$ dimensional hyperplane in a higher dimensional spacetime.
- Dynamical rather than rigid, because in the presence of gravity the hyperplane can fluctuate in shape and position as a dynamical object.
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  - Open strings (to which are associated gauge particles) end on the D-branes and are localised on them
  - Closed strings (corresponding to gravitons) are free to propagate in the bulk.
- The boundary conditions which localise the open string on the Dp-brane are Dirichlet conditions and hence the name Dirichlet brane or D brane. Since open strings are distinguished from the closed strings only by their end-points it is only the ends of the open strings that are confined to the branes.
- Consequently, a Dp-brane can be thought of as a flat hyperplane, to begin with, and then fluctuations of the shape of the brane being due to open string states.

- A new picture of the universe now emerges: the gauge particles of the SM live on a 3-brane, which resides with other branes in a higher dimensional spacetime. While the SM is confined to the brane, only gravity is free to wander over all spacetime including the bulk.

The ADD Model

- Assume that out four dimensional universe is a 3-brane on which the SM is localised and the gravitons are free to move around over the entire space-time.

- If the compactification radius of n extra dimensions is R, then
$$M_{\mathrm{P}}^2 = M_S^{n+2} R^n \ , \tag{2}$$
where M_S is the low-energy effective string scale.

- If R is large then M_S can be chosen to be of the order of a TeV i.e. hierarchically smaller than the Planck scale. For such a value of M_S, $R = 10^{32/n-19}$ m.

n	R (in m)
1	10^{13}
2	10^{-3}
3	4.5×10^{-9}
4	10^{-12}
5	2.5×10^{-13}
6	2.1×10^{-14}

- For $n = 2$ $R \sim 1$ mm, within the possible reach of ongoing gravitation experiments.

The View From the Braneless End

- Let us look at low-energy effective theory (rather than worry about the high-energy ineffective theory!!).
- Below the scale M_S the following effective picture emerges: After a toroidal compactification of the extra dimensions there are the Kaluza-Klein excitations of the graviton which correspond to a tower of spin-2, spin-1 and spin-0 excitations.
- The spin-1 modes do not couple to the energy-momentum tensor and their couplings to the SM particles in the low-energy effective theory are not important.
- The spin-2 Kaluza-Klein states manifest as an infinite tower of massive gravitons in the low-energy effective theory. There are also the spin-0 scalar excitations called the dilaton.
- Moreover, there are other light modes related to the brane dynamics. For example, there are the Y modes which are the massless Nambu-Goldstone bosons associated with the spontaneous breaking of translational invariance in the transverse directions. In the effective theory, however, some of these modes could acquire mass. Moreover, the Y modes couple to the SM matter only in pairs so their effects, compared to the graviton, are subleading.

- The effective description reduces to one where the gravitons in the bulk propagate in the flat background and couple to the SM fields which live on the brane via a (four-dimensional) induced metric $g_{\mu\nu}$.

- Starting from a linearized gravity Lagrangian in n dimensions, the four-dimensional interactions can be derived after a Kaluza-Klein reduction has been performed. The interaction of the SM particles with the graviton, $G_{\mu\nu}$, can be derived from the following Lagrangian:

$$\mathcal{L} = -\frac{1}{\bar{M}_P} G^{(j)}_{\mu\nu} T^{\mu\nu} \ , \tag{3}$$

  where j labels the Kaluza-Klein mode and $\bar{M}_P = M_P/\sqrt{8\pi}$, and $T^{\mu\nu}$ is the energy-momentum tensor.

Branes and Space-Time Symmetries

- In the non-gravitational limit, the 3-brane vacuum spontaneously breaks the d-dimensional Poincaré symmetry. Both translations transverse to the brane and the Lorentz transformations are broken yielding Nambu-Goldstone modes Y_m and $\theta_{\alpha a}$.
- The effective theory is invariant under the full d-dimensional Poincaré symmetry, the 4-dimensional Poincaré subgroup being linearly realized and the remaining symmetry transformations being non-linearly realized. The fact that the 4-dimensional theory respects the d-dimensional Lorentz invariance can be traced to the special couplings of the Nambu-Goto modes. Finally, gravity can be turned on by thinking of it as weakly gauging the spontaneously broken d-dimensional Poincaré group.

Graviton Production Processes

- The coupling of the gravitons to the SM particles is suppressed by $1/\bar{M}_P \Longrightarrow$ production of individual graviton states is hopelessly suppressed. But sum over the tower of KK excitations enhances the rate by M_P/M_S.
- Production of gravitons give rise to characteristic missing energy or missing p_T signatures at e^+e^- or hadron colliders resulting in bounds on M_S which are around 500 GeV to 1.2 TeV at LEP2 and around 600 GeV to 750 GeV at Tevatron (for n between 2 and 6).
- Gravitons produced in the core of supernovae in nucleon nucleon bremsstrahlung process can carry away energy from the core. Data on SN1987A from IMB and Kamiokande experiments imply $E \geq 2 \times 10^{53}$ ergs emmitted in the form of a neutrino flux $\Longrightarrow$ bounds on M_S of the order of 50 TeV for n=2 and about 2.5 TeV for n=3.
- Similar graviton production processes studied in the Sun and red giants yield bounds on M_S which are of the order of a TeV for $n = 2$.

Virtual Effects

- The KK excitations will also manifest in collider experiments through their virtual effects.
- The sum over the KK propagators is tricky because it is sensitive to unknown ultraviolet effects and is divergent for $n > 1$.
- Brane fluctuations, which are due to the Y modes, suppress the higher KK modes exponentially. Knowledge of the full high energy theory will help. (Revenge of the high-energy ineffective theory – Peskin).
- Naive approach is to regulate the divergence using a cut-off M'_S. Virtual processes yield bounds on M'_S whose relationship to M_S requires knowledge of the full theory.
- Virtual effects of graviton exchange in the processes listed below have been studied with the bounds on M_S:

  Dilepton production at Tevatron: $M_S =$ 950 GeV to 1100 GeV.

  $t\bar{t}$ production at Tevatron $M_S =$ 650 GeV while at the LHC $M_S =$ 4 TeV.

  Deep-inelastic scattering at HERA: $M_S =$ 550 GeV.

  Dijet production at the Tevatron: $M_S = 1.2$ TeV.

  Pair production of gauge bosons and fermions at LEP2: $M_S =$ 500 GeV – 1 TeV

Transplanckian Effects

- At the LHC, the collision energies are often in the trans-planckian domain.
- If the impact parameter in the collision between two partons is smaller than the n-dimensional Schwarzschild radius R, given by
$$R_S = \frac{1}{M_S}\left[\frac{M_{\rm BH}}{M_s}\right]^{\frac{1}{n+1}}\left[\frac{2^n \pi^{\frac{n-3}{2}}\Gamma(\frac{n+3}{2})}{n+2}\right]^{\frac{1}{n+1}} \tag{4}$$
then a black-hole maybe formed, if the entropy of the resulting system is sufficiently high.
- The production cross-section, given by the geometrical cross-section πR_S^2, can be huge at the LHC ($\sim$ 400 pb for M_S and $M_{\rm BH}$ of O(1 TeV)).
- The spin of the black hole is important so it may be more appropriate to consider Kerr solutions.
- The decay proceeds mainly through Hawking radiation and primarily into SM particles on the brane.
- Decay proceeds through three phases: a balding phase where the black hole loses hair associated with multipole moments, a spin-down phase where the angular momentum is shed, followed by a longer Schwarzschild phase. One ends up with large multiplicity events with hard jets and leptons.
- Hard processes taking place at short distances are suppressed because of the formation of an event horizon.

The Randall-Sundrum Model

- Out of an attempt to solve the hierarchy problem without introducing a large compactification radius came the model of Randall and Sundrum.

  In this model, the spacetime is five dimensional and strongly curved. The fifth dimension is compactified on a $\mathbf{S}^1/\mathbf{Z}^2$ orbifold. Two branes are located at the orbifold fixed points: a Planck brane at $\phi = 0$ and a TeV brane at $\phi = \pi$.

- The tension on the branes and the cosmological constant in the bulk are then fine-tuned to give a flat 4-dimensional geometry on the three-branes. The metric that guarantees is

$$ds^2 = e^{-KR_c\phi}\eta_{\mu\nu}dx^\mu dx^\nu + R_c^2 d\phi^2. \tag{5}$$

  where ϕ is the fifth dimension and R_c its the size which is assumed to be somewhat larger than the Planck length and K is a mass scale related to the curvature.

- The metric is $non-factorisable$ or $warped$. The warp factor acts as a conformal factor for the fields localised on the brane and mass factors get rescaled by this factor.

- So M_P could be 10^{19} GeV for the Planck brane located at $\phi = 0$ but gets rescaled to $M_P \exp(-KR_c\pi)$ for the TeV brane located at $\phi = \pi$. This exponential solves the hierarchy problem, because it is possible in this model to generate this enormous factor of 10^{15} by an exponential of order 30.

- Since the gauge hierarchy is achieved by a particular brane separation, a stabilisation mechanism is needed to obtain that. This is done by introducing a bulk scalar field to generate a stabilising potential. This is the radion which acquires a mass of O(Tev) and coupling 1/TeV.

- The radion couples to T^μ_μ and hence to massive particles. Interactions are very similar to that of the SM Higgs except that the vev is of O(TeV). Also the radion has anomalous couplings from the trace anomaly to gluons and photons.

More on Randall-Sundrum

- The interactions of the KK excitations with the observable particles on the TeV brane are given by the following Lagrangian:

$$\mathcal{L} = -\frac{1}{\overline{M}_P} T^{\mu\nu}(x) h^0_{\mu\nu}(x) - \frac{e^{\pi K R_c}}{\overline{M}_P} \sum_{n=1}^{\infty} T^{\mu\nu}(x) h^n_{\mu\nu}(x) \tag{6}$$

- The zero-mode of the KK tower couples very weakly (suppressed by $\overline{M}_P^{-1}$). The couplings of the massive KK states are enhanced by the exponential $e^{\pi K R_c}$ making the coefficient of electroweak strength. The Feynman rules that result from this Lagrangian are essentially the same as that for the ADD case.

- The equations of motion for the four-dimensional fields $h^n_{\mu\nu}(x)$ alongwith Einstein's equations, lead to a Bessel equation for the fields, the solutions for which are given as linear combinations of Bessel and Neumann functions.

- Continuity requirements of the solutions at the orbifold fixed points yield a simple relation for the masses M_n of the KK excitations in terms of the zeroes, x_n, of the Bessel function of order 1.

$$M_n = x_n K e^{-\pi K R_c} \equiv x_n m_0 \tag{7}$$

where m_0 sets the scale for the masses of the excitations.

- It is also expedient to define a parameter c_0 as

$$c_0 = \frac{e^{\pi K R_c}}{\overline{M}_P}\frac{m_0}{\sqrt{8\pi}} \tag{8}$$

The parameters m_0 and c_0 are the free parameters of the model.

Warped Phenomenology

- The distinctive features of the Randall-Sundrum model are as follows: There are no light KK modes. The constraints from real graviton production in current collider experiments do not apply.
- The astrophysical bounds also do not constrain this model because of the inhibition of the graviton production processes.
- In principle, higher energy collider experiments could produce individual graviton resonances and then individual modes can be identified by their decay products.
- The best probes for the RS model, however, are provided by virtual processes. For example, virtual effects in dilepton or diphoton production at Tevatron, fermion pair production at e^+e^- colliders or $t\bar{t}$ production at Tevatron.
- Radion phenomenology: Gluon fusion dominates the hadronic production cross-section for the radion. The best hope for the detection of the radion is via the gg initial state at the LHC and to look for decay into WW/ZZ final states.

Assorted Topics

- Off the Wall: SM particles also propagate in the extra dimensions of size TeV^{-1}. Model 1: Fermions are localised on the brane and the gauge bosons propagate in the bulk $\Longrightarrow$ KK excitations of the gauge bosons. In a 5-d model strong constraints from electroweak precision tests are obtained. Makes the collider searches for the excitations beyond the reach of present and linear colliders and only tractable at LHC. Model 2: Universal extra dimensions where all particles propagate in the bulk and no branes $\Longrightarrow$ momentum conservation in the extra dimension $\Longrightarrow$ KK number conservation in 4-d which helps dilute the electro-weak precision constraints. KK number conservation also implies these excitations can only be pair-produced thereby changing the collider signals.

- Unification: KK excitations of gauge bosons affects the running of gauge couplings. The logarithmic running of the couplings is replaced by a power law running, which leads to unification at very low scales.

- Neutrinos: A small mass of the neutrino conventionally suggests a mass scale of 10^{12} GeV. How does this fit in with a TeV scale theory? One solution uses a ν_R in the bulk which couples to the standard lepton doublet and the Higgs field on the brane via $h/\sqrt{M_S}\bar{L}\nu_R$. Once the Higgs develops a vev, the four dimensional mass terms look like

$$m\bar{\nu}_l(\nu_{0R} + \sqrt{2}\sum_{n=1}^{\infty}\nu_{nR}) \tag{9}$$

where $m \sim 10^{-4}$ eV if $M_S \sim 1$ TeV.

- The Power of Orbifolding: Orbifolds may well hold the key to solving some important problems like the proton decay problem or the doublet-triplet splitting problem. On a $Z - 2$ orbifold, for example, all fields may be decomposed into even and odd functions of the extra dimensional co-ordinates. By properly assigning the right parity to these fields one can get around the proton decay or doublet-triplet splitting problem.