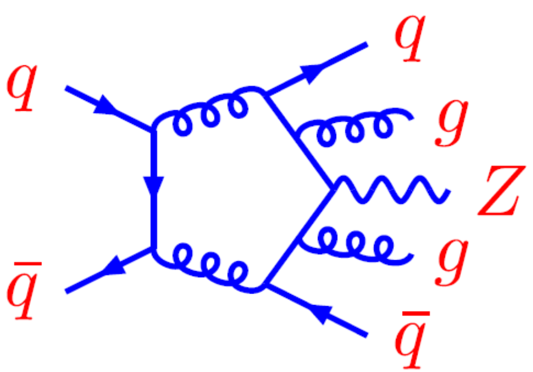
Status of NLO Multileg
“New Approaches”

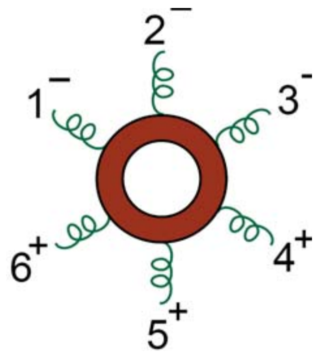


Lance Dixon, SLAC

Les Houches
June 12, 2007

Computational problem

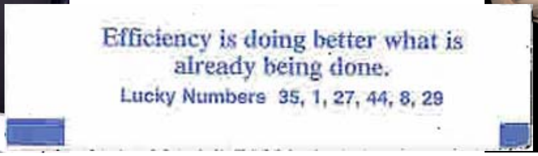
~ 10,000 Feynman diagrams →
Most have complex dependence on kinematic variables



Choice of hardware?

Silicon-based

Carbon-based



Not either/or. Question is whether studying detailed properties of amplitudes can lead to more efficient (Si-based) evaluation

State of the art

Benchmark process: 6-gluon amplitude, *gg* → *gggg*

- evaluated **"semi-numerically"** using Feynman-diagram-based representations and loop-integral reductions on the fly

Ellis, Giele, Zanderighi, hep-ph/0602185

- and **analytically**

...; Bedford, Brandhuber, Spence, Travaglini, hep-th/0412108;
Britto, Buchbinder, Cachazo, Feng, hep-ph/0503132;
Britto, Feng, Mastrolia, hep-ph/0602178;
Berger, Bern, LD, Forde, Kosower, hep-ph/0604195, hep-ph/0607014;
Xiao, Yang, Zhu, hep-ph/0607017

- Advantage of semi-numerical approach: flexibility (e.g., *g* → *q*)
- Advantage of analytical approach: speed of evaluation
- 30 msec (for all but $(-+-+-+)$ & $(--+-++)$) vs. 9 sec per phase space point
- Speed important for Monte Carlo phase-space integrations

State of the art (cont.)

• It is also possible to proceed **fully numerically**,
e.g. 6-photon 1-loop amplitude, *gg → gggg*

Nagy, Soper, hep-ph/0308127, hep-ph/0610028

• and pp → *ZZZ* at NLO

Lazopoulos, Melnikov, Petriello, hep-ph/0703273

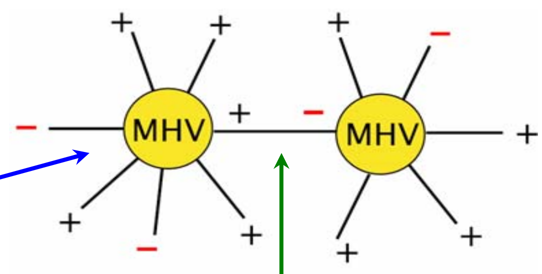
• Requires paying attention to the singularity structure of the one-loop integrand

• Important to see if this technique can be extended to cases with additional partons in the final state

Twistor string theory
→ renewed interest in studying analytic properties of amplitudes

Led to MHV rules:

Witten (2003); Cachazo, Svrcek, Witten (2004)

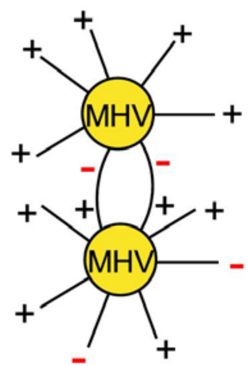


off-shell MHV (Parke-Taylor) amplitudes

scalar propagator, $1/p^2$

Efficient alternative to Feynman rules for QCD trees

Can also give partial information about QCD loops



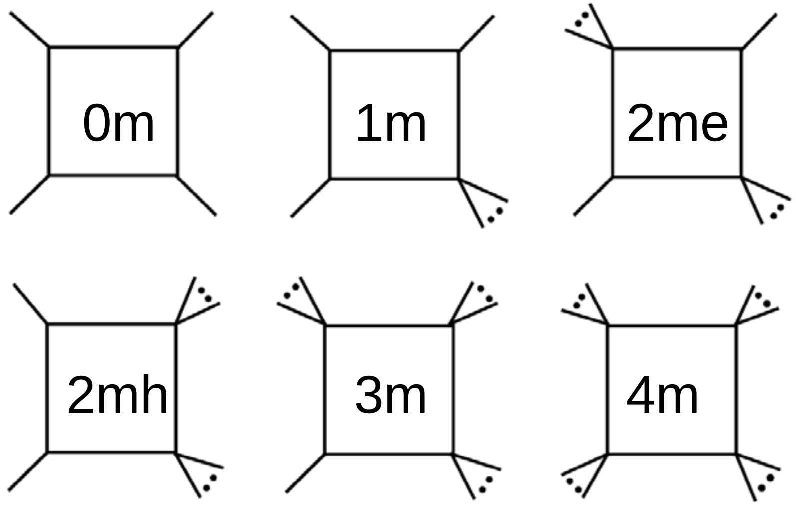
Brandhuber, Spence, Travaglini, hep-th/0407214, hep-th/0410280;
Bedford, Brandhuber, Spence, Travaglini, hep-th/0412108

Similar to that provided by unitarity

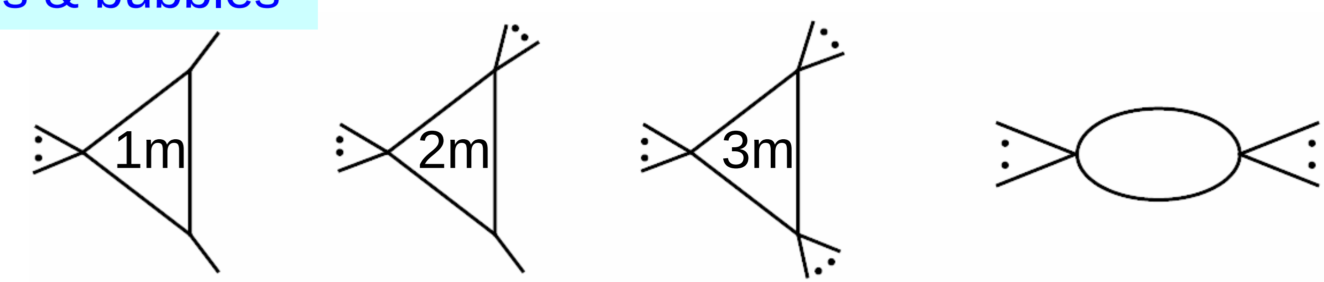
Bern, LD, Dunbar, Kosower, hep-ph/9403226, hep-ph/9409265

General decomposition of 1-loop amplitudes

Boxes



Triangles & bubbles



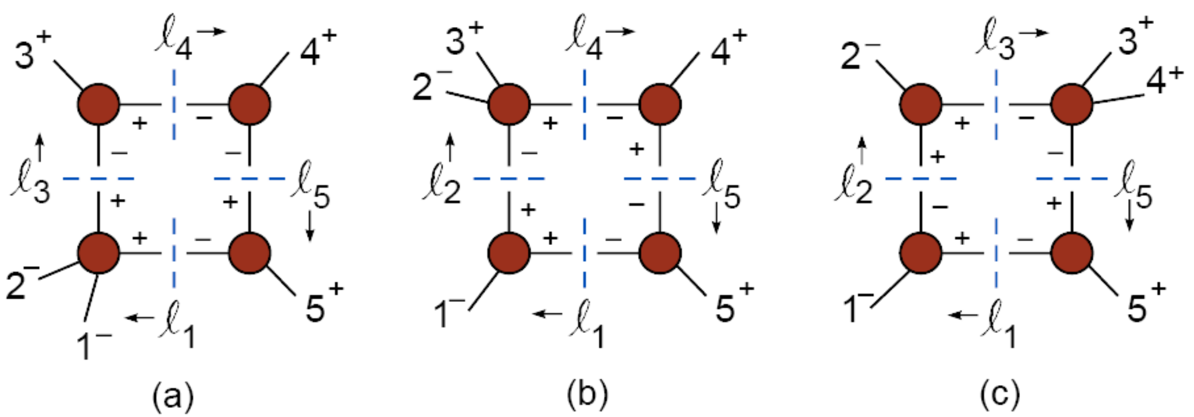
"Just" need to work out the coefficients of each integral

(Generalized) unitarity can be used to determine coefficients of integrals

- Boxes determined by quadruple cuts.
- Four constraints determine all four components of loop momenta

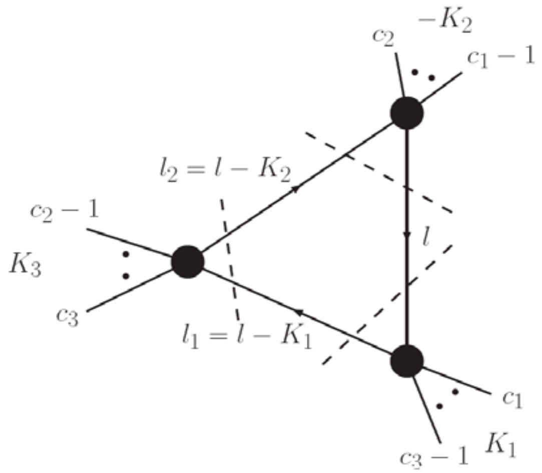
→ coefficients obtained by simple substitution

Britto, Cachazo, Feng, hep-ph/0412103



(Generalized) Unitarity (cont.)

• Triangles are determined by triple cuts, which have one loop-momentum component undetermined.
• Also must "project out" boxes participating in the same cut. Similar story for bubbles.
• A couple of methods proposed recently for doing this seem to work pretty well.



Ossola, Papadopoulos, Pittau, hep-ph/0609007;
Mastrolia, hep-th/0611091;
Forde, 0704.1835 [hep-ph]

Rational Parts

- Unitarity cuts are simplest to evaluate in four dimensions, because of many simplifications from helicity basis for intermediate states.
- However, in this case rational parts are missed.
- Three basic ways proposed to recover rational parts:
  - Feynman diagrams (many don't contribute)

    Xiao, Yang, Zhu, hep-ph/0607017; Binoth, Guillet, Heinrich, hep-ph/0609054
  - Unitarity in $D = 4\text{-}2\varepsilon$ Bern, Morgan, hep-ph/9511336;

    Bern, LD, Kosower, hep-ph/9602280;

    Bern, LD, Dunbar, Kosower, hep-ph/9611127;

    Brandhuber, McNamara, Spence, Travaglini, hep-th/0506068;

    Anastasiou, Britto, Feng, Kunszt, Mastrolia, hep-ph/0609191, hep-ph/0612277;

    Britto, Feng, hep-ph/0612089
  - Recursive approach

    Bern, LD, Kosower, hep-th/0501240, hep-ph/0505055, hep-ph/0507005;

    Berger, Bern, LD, Forde, Kosower, hep-ph/0604195, hep-ph/0607014;

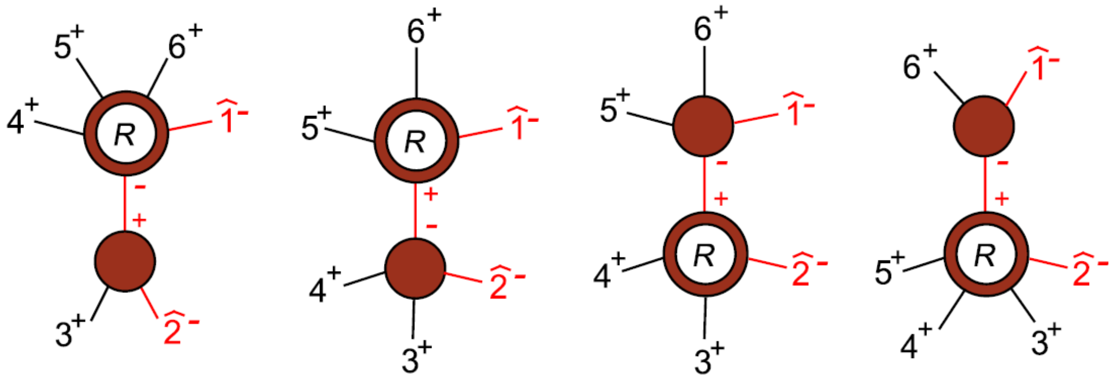
    Berger, Del Duca, LD, hep-ph/0608180 [*Hgggg*];

    Badger, Glover, Risager, arXiv:0704.3914 [hep-ph] [*Hgggg*];
- All three methods could use more development and/or automation

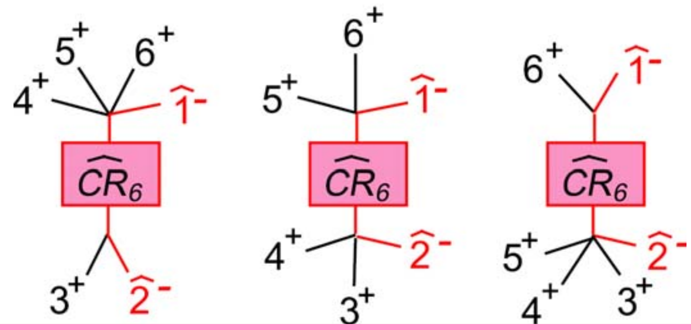
Example of recursive diagrams

For rational part of $A_6^{1-\text{loop}}(1^-,2^-,3^+,4^+,5^+,6^+)$

recursive:



overlap:



7 in all

Compared with 1034 1-loop Feynman diagrams (color-ordered)

But not fully worked out yet for general helicity configurations

Relating "new" and "standard" approaches

Mostly at tree level so far

- Derivation of MHV rules using non-local changes of variables

Gorsky, Rosly, hep-th/0510111; Mansfield, hep-th/0511264;
Ettle, Morris, hep-th/0605121

- MHV-like "scalar" rules for QCD + massive quarks, derived directly from field theory

Schwinn, Weinzierl, hep-th/0503015

- BCFW recursion relations by rearranging Feynman diagrams

Draggiotis, Kleiss, Lazopoulos, Papadopoulos, hep-ph/0511288

especially simple using "largest-time" equation in "space-cone" gauge

Vaman, Yao, hep-ph/0512031; Chalmers, Siegel, hep-th/9801220

Conclusions

- New computational approaches to tree and loop amplitudes in QCD stimulated (directly or indirectly) by twistor string theory
- Also related to older methods using unitarity, factorization, special gauges
- Some new loop-level helicity amplitudes for LHC applications have been found by the new techniques, but further development and/or automation is certainly required, especially for the rational parts
- Of course incorporating such amplitudes into NLO parton-level programs using e.g. Catani-Seymour dipole or antenna subtraction methods will be a challenge, as will interfacing to an “NLO MC”.
- **Multiple workshops in 2007** intended to bring proponents of “standard” and “new” techniques together for cross-fertilization:

Les Houches **Durham Twistor Workshop** **Galileo Institute**

- Hope the progress will continue at a pace to allow us to be ready for the flood of LHC data!

Some discussion topics for Session I

Wed. June 13

a.m.	Recursive methods (Stefan Weinzierl)
p.m.	Extracting integral coefficients (Roberto Pittau, Walter Giele)

Fri. June 15

a.m.	Semi-numerical methods (Thomas Binoth, Keith Ellis)
p.m.	Fully numerical methods (Yoshihasa Kurihara, Zoltan Nagy)

Mon. June 18

a.m. & p.m.	Matching parton showers to (N)LO matrix elements (Dave Soper, Steffen Schumann, Henrik Nilsen,…)

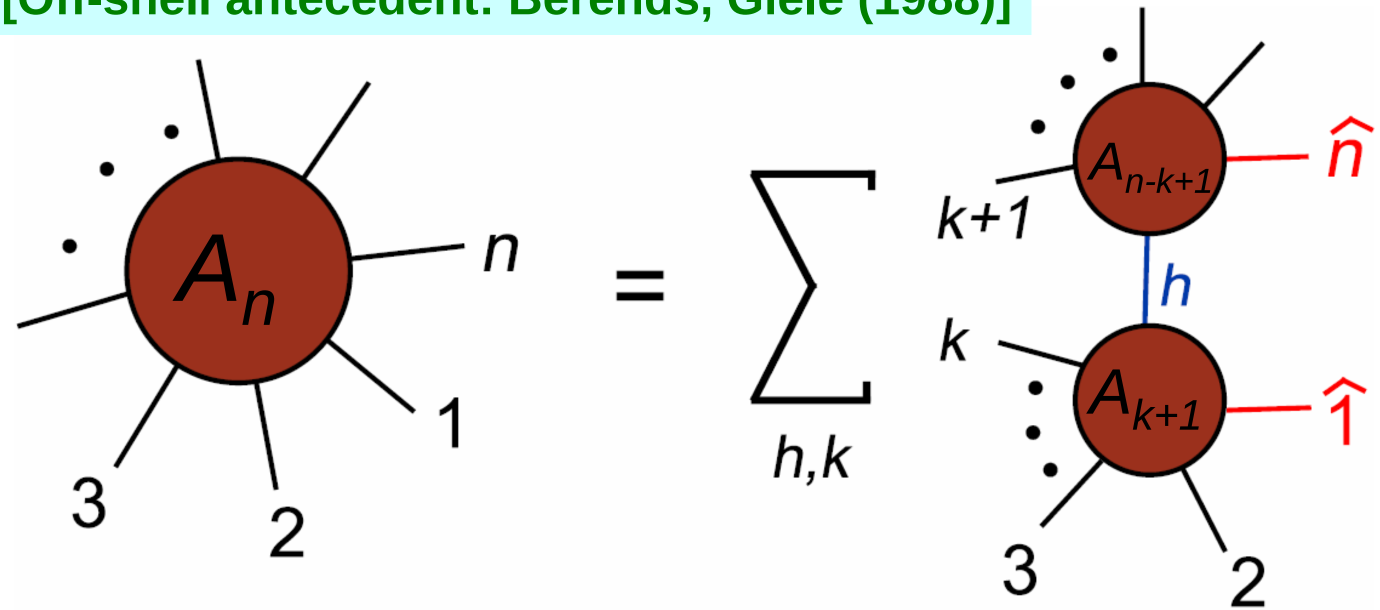
Extra slides

Recursive approach follows tree-level BCF(W) approach

On-shell recursion relations

Britto, Cachazo, Feng, hep-th/0412308

[Off-shell antecedent: Berends, Giele (1988)]



A_{k+1} and A_{n-k+1} are on-shell tree amplitudes with fewer legs, evaluated with momenta shifted by a **complex** amount

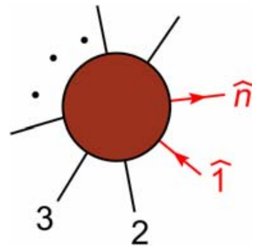
Trees are recycled into trees!

Proof of on-shell recursion relations

Britto, Cachazo, Feng, Witten, hep-th/0501052

Simple, general: Residue theorem + factorization

how amplitudes "fall apart" in degenerate kinematic limits

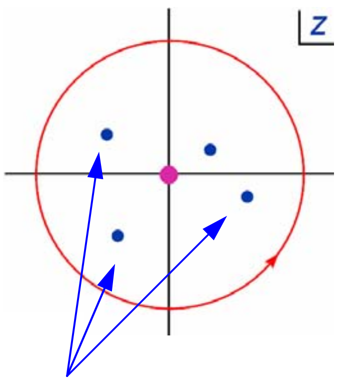


Inject complex momentum at leg 1, remove it at leg *n.*

$$k_1 = \lambda_1 \tilde{\lambda}_1 \ \to \ (\lambda_1 + z\lambda_n)\tilde{\lambda}_1 \quad \Rightarrow \quad A(0) \ \to \ A(z)$$

$$k_n = \lambda_n \tilde{\lambda}_n \ \to \ \lambda_n(\tilde{\lambda}_n - z\tilde{\lambda}_1)$$

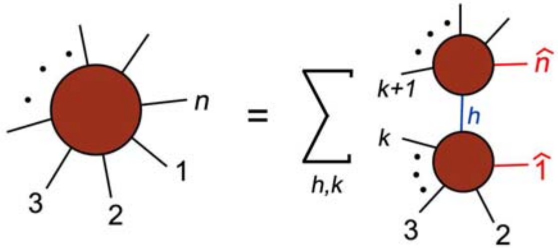
degenerate limits ⇔ poles in *z*



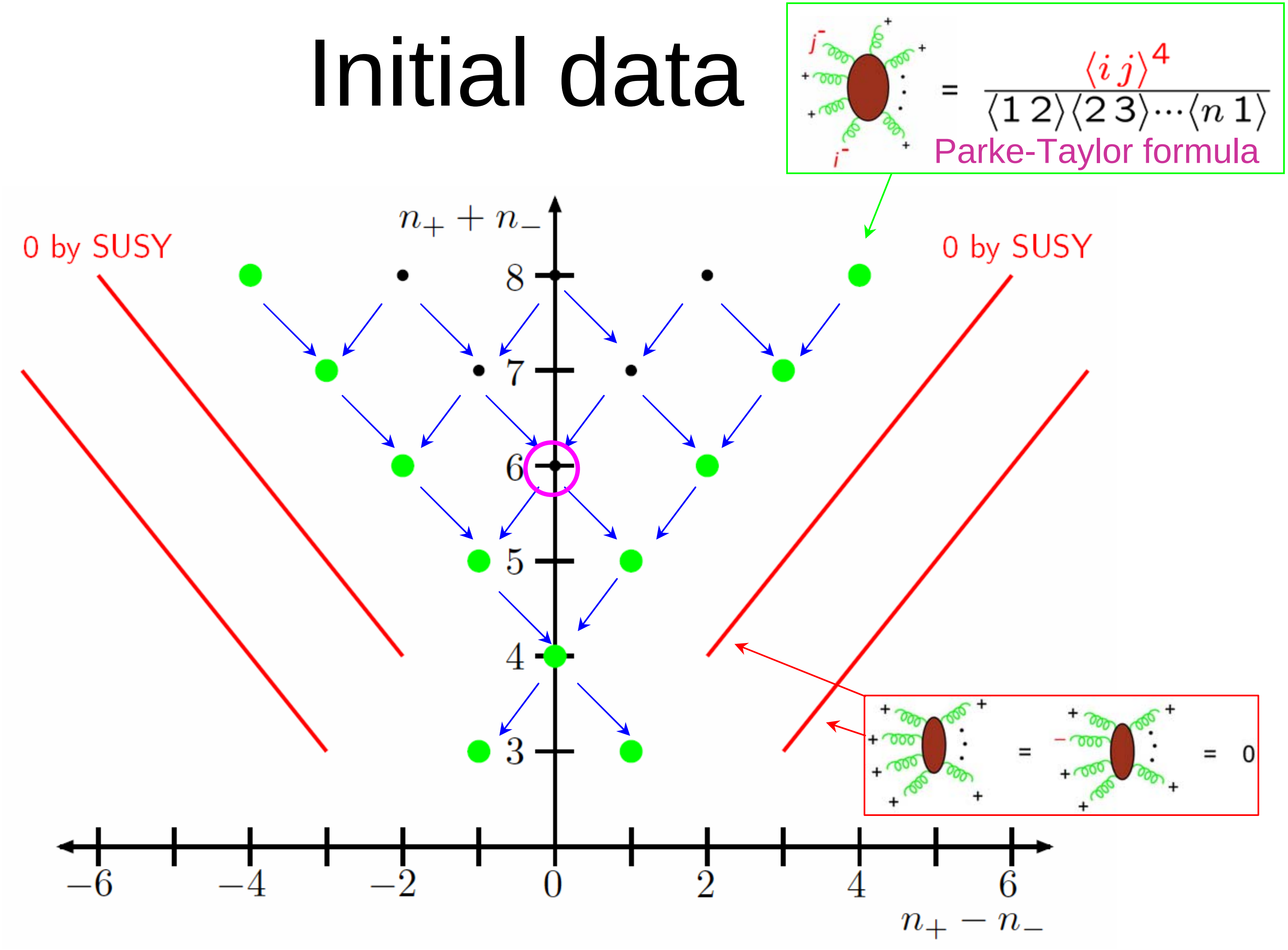
Cauchy: If $A(\infty) = 0$ then

$$0 = \frac{1}{2\pi i}\oint dz \frac{A(z)}{z} = A(0) + \sum_k \text{Res}\left[\frac{A(z)}{z}\right]\Big|_{z=z_k}$$

residue at z_k
= [k^{th} term in relation]



Initial data



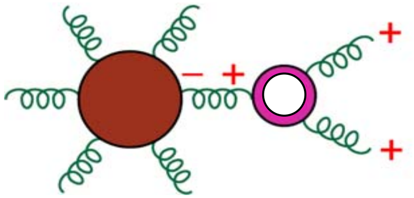
On-shell recursion at one loop

Bern, LD, Kosower, hep-th/0501240, hep-ph/0505055, hep-ph/0507005

- **Same techniques** work for one-loop QCD amplitudes – much harder to obtain by other methods than are trees.

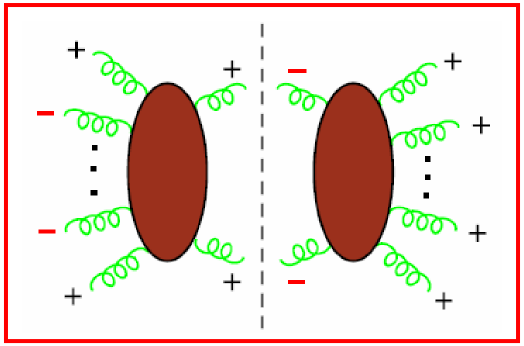
- **New features** arise compared with tree case

1. **different collinear behavior** of loop amplitudes (double poles in z)



2. **branch cuts** – but these can be determined efficiently using (generalized) unitarity

Trees recycled into loops!



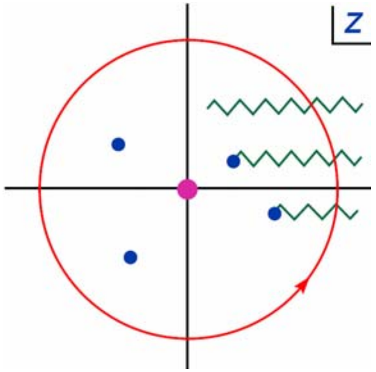
Bern, LD, Kosower, hep-ph/9403226, hep-ph/9708239; Britto, Cachazo, Feng, hep-th/0412103

Loop amplitudes with cuts

Generic analytic properties of shifted 1-loop amplitude, $A_n(z)$

Cuts and poles in *z*-plane:

$$\ln(s_{23}) \Rightarrow \ln[(\langle 2\,3\rangle + z\langle 1\,3\rangle)[3\,2]]$$



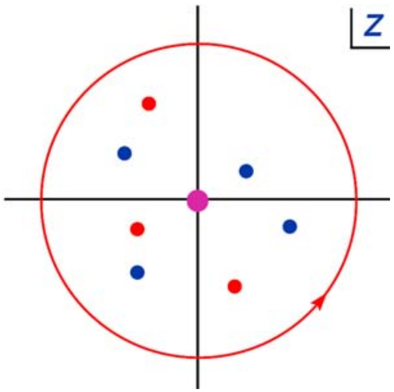
But if we know the cuts (*via* unitarity in D=4), we can subtract them: $R_n \equiv A_n - C_n$

rational part — full amplitude — cut-containing part

Shifted rational function $R_n(z) = A_n(z) - C_n(z)$

has no cuts, but has spurious poles in *z* because of C_n:

$$C_n \longrightarrow \frac{\ln(r) + 1 - r}{(1-r)^2} \longleftarrow R_n$$



Rational functions in loop amplitudes

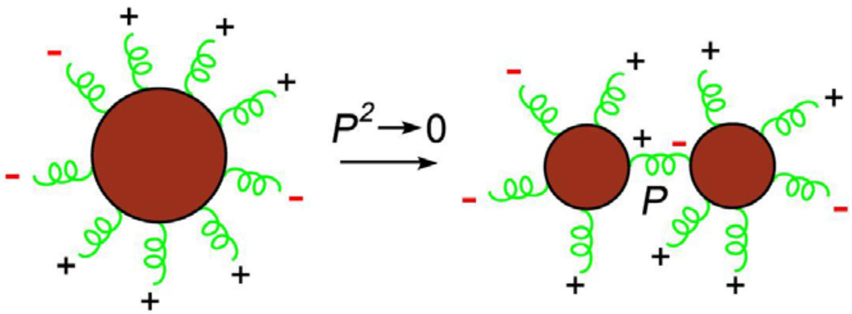
- Compute cuts using unitarity
- Leaves additive rational function ambiguity
- Determined using
  - tree-like recursive diagrams, plus
  - simple "overlap diagrams"
- No loop integrals required in this step
- Bootstrap rational functions from cuts and lower-point amplitudes
- Method tested on 5-point amplitudes, used to get new QCD results:
  - all *n*-gluon 1-loop MHV amplitudes $(- + \cdots + - + \cdots +)$
  - "split" helicity amplitudes $(- - \cdots - + + \cdots +)$

  Forde, Kosower, hep-ph/0509358;
  Berger, Bern, LD, Forde, Kosower, hep-ph/0604195, hep-ph/0607014
- To do: Generalize method to all helicity configurations, and to processes on the "realistic NLO wishlist".

Revenge of the Analytic *S*-matrix

Reconstruct scattering amplitudes **directly** from **analytic properties**

- Poles



- Branch cuts