## Out-of-Acceptance Treatment: Concept



Measurement bins
There are $2 n$ POls ( $l_{i}$ and $O_{i}$ for each measurement bin), but only $n$ observables (the bin yield $N_{1}$ ). So we need some assumption about the $\mathrm{O}_{\mathrm{i}}$ in order to measure the $I_{1}$ :

- Option 1: All the $\mathrm{O}_{\mathrm{i}}$ are fully correlated to each other $\Rightarrow$ all scale with a global O parameter - not constrained by data, so fix to the SM.
- Option 2: Each $\mathrm{O}_{\mathrm{i}}$ is fully correlated to the corresponding $\mathrm{I}_{\mathrm{i}}$.
- Option 3: All the $\mathrm{O}_{\mathrm{i}}$ are fully correlated to each other, and to $\Sigma_{i} I_{i}$


## Out-of-Acceptance: Math

Measured bin yields are $N_{i}=\Sigma_{j} m_{i j} I_{j}+\Sigma_{j} f_{i j} O_{j}$ where
$-I_{i}=L . \sigma_{i d i}^{i d}$ : Fiducial contribution
$-O_{i}=L . \sigma^{\text {out-of-fid }}=\mathrm{L}\left(\sigma_{i}^{\text {tot }}-\sigma_{i}^{\text {fd }}\right)$ : Out-of-fiducial contribution

- $m_{i j}$ : migration coefficients for the fiducial contributions into reco bins
- $f_{i j}$ : contamination coefficients for non-fiducial contributions
- Option 1 : We assume $O_{i}=\alpha_{i} O^{\text {sM }}$, with $\alpha_{i}=O_{i}^{\text {SM }} / O^{\text {SM }}$ taken from SM MC

$$
\Rightarrow \mathrm{N}_{\mathrm{i}}=\Sigma_{\mathrm{j}} \mathrm{~m}_{\mathrm{ij}} \mathrm{I}_{\mathrm{j}}+\Sigma_{\mathrm{j}} \mathrm{f}_{\mathrm{ij}} \alpha_{\mathrm{j}} \mathrm{O}^{s \mathrm{M}}
$$

- Option 2 : We assume $O_{i}=\beta_{i} l_{i}$, with $\beta_{i}=O_{i}^{S M} / l_{i}^{S M}$ taken from SM MC

$$
\Rightarrow N_{i}=\Sigma_{j}\left(m_{i j}+\beta_{i} f_{i j}\right) l_{j}
$$

- Option 3 : We assume $O_{i}=\gamma_{i} \Sigma_{j} I_{j}$, with $\gamma_{i}=O_{i}^{S M} / \Sigma_{j}{ }_{j}^{\text {SM }}$ taken from SM MC $\Rightarrow N_{i}=\Sigma_{j} m_{i j} I_{j}+\Sigma_{j} f_{i j} \gamma_{j} \Sigma_{k} I_{k}=\Sigma_{j}\left(m_{i j}+\Sigma_{k} f_{i k} \gamma_{k}\right) l_{j}$


## What can go wrong

- Consider a 1-bin analysis with a spectacularly bad fiducial selection, $\mathrm{m}=0.1$ and $\mathrm{f}=0.9$ $\Rightarrow \mathrm{N}=0.1 \mathrm{I}+0.9 \mathrm{O}$; assume $\mathrm{I}_{\mathrm{SM}} \sim \mathrm{O}_{\mathrm{SM}}$ and consider
- Case $A: I_{\text {true }}=I_{\text {SM }}$ and $O_{\text {true }}=O_{S M}$
- Case B: $I_{\text {true }}=I_{S M}$ and $O_{\text {true }}=10 O_{S M}$
- Case C: $I_{\text {true }}=10 I_{S M}$ and $O_{\text {true }}=10 O_{S M}$
- Option 1: $\mathrm{N}=0.1 \mathrm{I}+0.9 \mathrm{O}_{\mathrm{SM}}$.
- Case A
- $\mathrm{I}=\left(\mathrm{N}-0.9 \mathrm{O}_{\text {SM }}\right) / 0.1=\left(0.1 \mathrm{I}_{\text {SM }}+0.9 \mathrm{O}_{\text {SM }}-0.9 \mathrm{O}_{\text {SM }}\right) / 0.1=\mathrm{I}_{\text {SM }}=\mathrm{I}_{\text {true }}$ accurate
- $\delta \mathrm{l}=\mathrm{sqrt}(\mathrm{N}) / 0.1$ : error larger due to low purity
- Case B: $\mathrm{I}=\mathrm{I}_{\text {SM }}+81 \mathrm{O}_{\text {SM }} \sim 82 \mathrm{I}_{\text {true }}$, biased
- Case C: $\mathrm{I}=10 \mathrm{I}_{\text {SM }}+81 \mathrm{O}_{\mathrm{SM}} \sim 91 \mathrm{I}_{\text {true }}$ biased
- Option 2: $\mathrm{N}=\left(0.1+0.9 \mathrm{O}_{\mathrm{SM}} / /_{\mathrm{SM}}\right)$ (Option 3 is equivalent in this 1-bin case)
- Case A:
- $\mathrm{I}=\mathrm{N} /\left(0.1+0.9 \mathrm{O}_{\text {SM }} / I_{\text {SM }}\right)=I_{\text {SM }}=I_{\text {true }}$ accurate
- $\delta \mathrm{l}=\operatorname{sqrt}(\mathrm{N}) /\left(0.1+0.9 \mathrm{O}_{\mathrm{SM}} / \mathrm{I}_{\mathrm{SM}}\right)$, error $\sim$ insensitive to purity
- Case B: $I=\left(0.1 I_{S M}+9 \mathrm{O}_{S M}\right) /\left(0.1+0.9 \mathrm{O}_{\text {SM }} / I_{\text {SM }}\right) \sim 10 I_{\text {true }}$ biased
- Case C: $I=\left(1 I_{S M}+9 \mathrm{O}_{\mathrm{SM}}\right) /\left(0.1+0.9 \mathrm{O}_{\mathrm{SM}} / I_{\mathrm{SM}}\right)=I_{\text {true }}$ accurate

