DM production via freeze-in

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1 Idea

Here I write in as much detail as possible the computation of relic abundance of dark matter via freeze-in mechanism. We assume first a simple model containing the following vertex:

$$\mathcal{L} \supset y_\chi (\chi \cdot \bar{f}_L) S + h.c. \tag{1}$$

where χ is a fermion doublet whose quantum numbers are contrated with a SM fermion f, and S is the DM candidate, a real scalar.

2 Production via decay

The process producing DM here is $M \to BS$, where M is the mother particle, B a particle belonging to the visible thermal bath, and S is the dark matter. Being so, the Boltzmann equation for the DM can be written as:

$$\begin{aligned} \dot{n}_S + 3Hn_S &= \text{production} - \text{annihilation} \qquad (2) \\ &= \int \frac{d^3p_M}{(2\pi)^3 2E_M} \frac{d^3p_B}{(2\pi)^3 2E_B} \frac{d^3p_S}{(2\pi)^3 2E_S} (2\pi)^4 \delta^{(4)}(P_M - P_B - P_S) |\mathcal{M}|^2 \\ &\times \left[f_M(1 \pm f_B)(1 \pm f_S) - f_B f_S (1 \pm f_M) \right] , \end{aligned}$$

where n_S is the DM number density, $H = \dot{a}/a$ the Hubble parameter, $p_i(P_i)$ the 3-momentum (4-momentum) of particle i, and E_i its energy; $\mathcal{M}$ the amplitude of the process, and f_i the distribution function. In case of fermions (bosons), $\pm$ becomes $-(+)$, corresponding to Fermi-Dirac blocking (Bose-Einstein enhancement).

Assumption #1: initial density of DM particles is zero, such that the annihilation term can be neglected. This is reasonable when the production rate is so slow that you never have sufficient DM particles in order for the annihilation term to be non-negligible. With this, (2) reduces to:

$$\begin{aligned} \dot{n}_S + 3Hn_S &= \int \frac{d^3p_M}{(2\pi)^3 2E_M} \frac{d^3p_B}{(2\pi)^3 2E_B} \frac{d^3p_S}{(2\pi)^3 2E_S} (2\pi)^4 \delta^{(4)}(P_M - P_B - P_S) |\mathcal{M}|^2 \\ &\times f_M(1 \pm f_B)(1 \pm f_S) \ . \end{aligned} \tag{3}$$

This is the equation we want to solve next. On the one hand we have the expression for the width Γ of the $M \to BS$ process, for an -a priori- boosted particle :

$$d\Gamma_{\rm lab} = \frac{(2\pi)^4}{2E_M}|\overline{\mathcal{M}}|^2 d\Phi, \quad d\Phi = \frac{d^3p_B}{(2\pi)^3 2E_B}\frac{d^3p_S}{(2\pi)^3 2E_S}\delta^{(4)}(P_M - P_B - P_S) \tag{4}$$

Note that the averaged amplitude squared is not what enters into eq.(3), being $|\mathcal{M}|^2 = \frac{1}{g_M}|\overline{\mathcal{M}}|^2$. We can thus rewrite eq.(3) as:

$$\dot{n}_S + 3Hn_S = g_M \int \frac{d^3p_M}{(2\pi)^3} f_M \Gamma_{\rm lab} = g_M \Gamma m_M \int \frac{d^3p_M}{(2\pi)^3}\frac{f_M}{E_M} \ . \tag{5}$$

Here Γ is the usual (CM frame) width.

Assumption #2: In expr.(5) we have neglected Pauli-blocking (Bose-Einstein enhancement) effects, which implies that $f_{B,S} \ll 1$ (see expr.3). For the DM this is absolutely reasonable, whereas for the bath particle B this is less clear[1]. In any case this is the usual practice, starting with the original freeze-in reference[1].

Assumption #3: For the distribution function f_M we will assume a Maxwell-Boltzmann (MB) shape, $f_M = e^{-(E_M-\mu_M)/T}$. The use of MB distribution means two things: a) we are neglecting the statistics of the particle M (fermion or boson), and b) we are assuming that M follows an equilibrium distribution. As for a), this is reasonable for $E \gg T$ (the case we will consider below, as you will see). Concerning b), this is less clear, but again, this is the usual practice (cf. [1],[3]), and the corrections to that are work in progress[2]. On top of that, we will neglect the chemical potential μ_M, such that $f_M \approx e^{-E_M/T}$. This is equivalent to say that the change on the total internal energy of the system (visible bath plus M plus S) by removing/introducing one M particle is negligible, which is not unreasonable. With these assumptions, we are left with:

$$\dot{n}_S + 3Hn_S = \frac{1}{2\pi^2} g_M \Gamma m_M^2 T K_1(m_M/T) \ , \tag{6}$$

where $K_1(x)$ is the modified Bessel function of the second kind. The LHS of expr.(7) can be re-expressed as: $\dot{n}_S + 3Hn_S = \mathbf{s}HT\frac{dY_S}{dT}$, where $\mathbf{s}$ is the entropy density, and $Y_S = n_S/\mathbf{s}$ is the comoving number density (or yield) of the DM. So we have:

$$Y_S = -\frac{1}{2\pi^2} g_M \Gamma m_M^2 \int_{T_R}^{T_0} dT \frac{K_1(m_M/T)}{\mathbf{s}H} \ , \tag{7}$$

where T_R is the reheating temperature and T_0 the temperature today. The entropy is $\mathbf{s} = (2\pi^2/45)g_*^s T^3$.

Assumption #4: Considering that the relevant DM production happens at a radiation-dominated era, we have $H \approx 1.66\sqrt{g_*}T^2/M_{\rm Pl}$, where $M_{\rm Pl}$ is the Planck mass. This should be refined if we start populating DM before reheating time, but in any case if most of

[1] A solution taking into account these effects will appear soon [2].

the DM will be created towards the end, where $T \ll T_R$, then indeed this is a radiation-dominated period.

The final expression for the yield is thus:

$$Y_S \approx \frac{45 M_{\rm Pl}}{4\pi^4 \cdot 1.66} \frac{g_M}{m_M^2} \Gamma \int_{m_M/T_R}^{m_M/T_0} dx \; x^3 \frac{K_1(x)}{g_*^s(x)\sqrt{g_*(x)}} \tag{8}$$

The link between today's relic abundance and yield of DM is:

$$\Omega = \frac{\rho_{0,S}}{\rho_c} \approx \frac{m_S Y_S}{3.6 \times 10^{-9}{\rm GeV}} \; . \tag{9}$$

This implies:

$$Y_S \approx 4.3 \times 10^{-10} \left(\frac{\Omega h^2}{0.12}\right) \left(\frac{\rm GeV}{m_S}\right) \; . \tag{10}$$

2.1 Example#1

Assuming that we have an instantaneous decay at $T \ll m_M$, then M is at rest and $\Gamma \approx y_\chi^2 M$, where we have neglected phase-space, i.e. $m_M \gg m_B, m_S$, and y_χ is the coupling of the $M - B - S$ interaction. The lifetime of M would then be:

$$\tau_M = 2 \times 10^{-14} \text{ cm} \left(\frac{1}{y_\chi^2}\right) \left(\frac{\rm GeV}{m_M}\right) \; . \tag{11}$$

For the benchmark $m_M = 1$TeV, $y_\chi = 5 \times 10^{-10}$ and $m_S = 1$ GeV, we get that $\tau_M \approx 80$ cm. For those, a reheating temperature $T_R = 47.5$ GeV gives the correct relic abundance according to (10).

References

[1] L. J. Hall, K. Jedamzik, J. March-Russell and S. M. West, JHEP **1003** (2010) 080 doi:10.1007/JHEP03(2010)080 [arXiv:0911.1120 [hep-ph]].

[2] Belanger, Goudelis, Pukhov and Zaldivar. In preparation.

[3] R. T. Co, F. D'Eramo, L. J. Hall and D. Pappadopulo, JCAP **1512** (2015) no.12, 024 doi:10.1088/1475-7516/2015/12/024 [arXiv:1506.07532 [hep-ph]].