Higgs boson pair production in gluon fusion at full NLO

Gudrun Heinrich

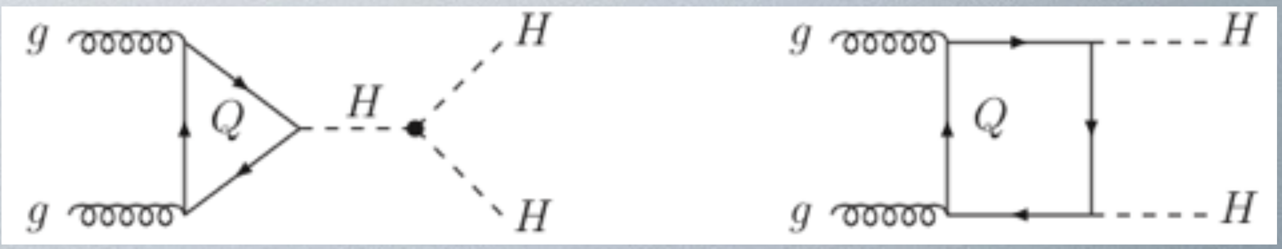
Max Planck Institute for Physics, Munich

Les Houches, June 10, 2017

MAX-PLANCK-GESELLSCHAFT

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

gg to HH



LO with full heavy quark mass dependence

Glover, van der Bij ’88, Plehn, Spira, Zerwas ’96

$m_t \to \infty$ limit (“HEFT”):

Note:

HEFT strictly valid only for $\sqrt{\hat{s}} \ll 2m_t$

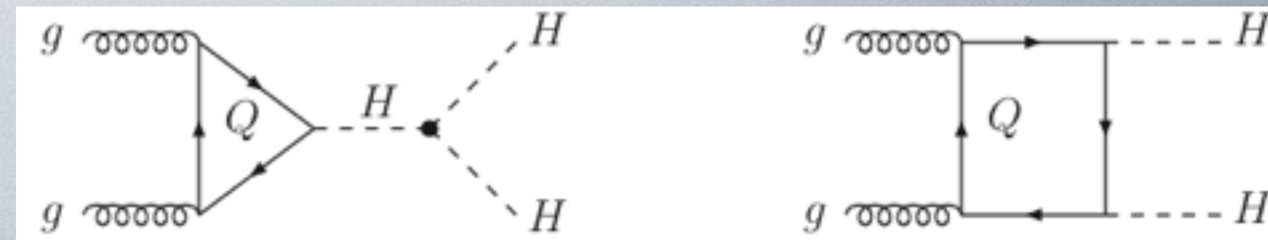
HH production threshold: $2m_H < \sqrt{\hat{s}}$

$\Rightarrow$ validity of HEFT limited to $250\,\mathrm{GeV} < \sqrt{\hat{s}} < 340\,\mathrm{GeV}$

gg to HH

LO with full heavy quark mass dependence

Glover, van der Bij '88, Plehn, Spira, Zerwas '96

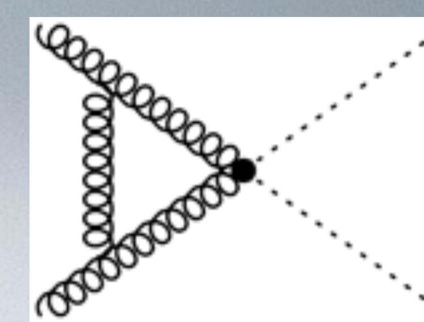


$m_t \to \infty$ limit ("HEFT"):

Note:

HEFT strictly valid only for $\sqrt{\hat{s}} \ll 2m_t$
HH production threshold: $2m_H < \sqrt{\hat{s}}$
$\Rightarrow$ validity of HEFT limited to $250\,\text{GeV} < \sqrt{\hat{s}} < 340\,\text{GeV}$

"Born-improved HEFT": rescale by $\mathcal{M}^{LO}(m_t)/\mathcal{M}^{LO}_{\text{HEFT}}$



NLO in Born-improved HEFT Dawson, Dittmaier, Spira '98 (**HPAIR**) $K \simeq 2$

- **supplemented with $1/m_t$ expansion:** $(\pm 10\%)$

  Grigo, Hoff, Melnikov, Steinhauser '13, '15 ; Degrassi, Giardino, Gröber '16

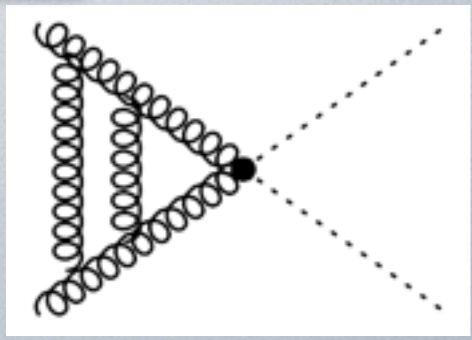
- **full mass dependence in NLO real radiation ("FTapprox")** **-10%**

  Frederix, Hirschi, Mattelaer, Maltoni, Torrielli, Vryonidou, Zaro '14; Maltoni, Vryonidou, Zaro '14

MAX-PLANCK-GESELLSCHAFT

Max-Planck-Institut für Physik (Werner-Heisenberg-Institut)

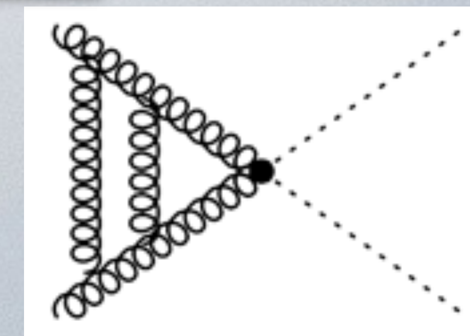
gg to HH



NNLO in $m_t \to \infty$ limit: **+20%**

- **total xs NNLO** De Florian, Mazzitelli '13
- **including all matching coefficients** Grigo, Melnikov, Steinhauser '14
- **supplemented with $1/m_t$ expansion:** Grigo, Hoff, Steinhauser '15
- **soft gluon resummation NNLL matched to NNLO** De Florian, Mazzitelli '15 **+9%**
- **differential NNLO** De Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev '16

gg to HH



NNLO in $m_t \to \infty$ limit: **+20%**

- **total xs NNLO** De Florian, Mazzitelli '13
- **including all matching coefficients** Grigo, Melnikov, Steinhauser '14
- **supplemented with $1/m_t$ expansion:** Grigo, Hoff, Steinhauser '15
- **soft gluon resummation NNLL matched to NNLO** De Florian, Mazzitelli '15 **+9%**
- **differential NNLO** De Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev '16

NLO calculation with full top mass dependence

Borowka, Greiner, GH, Jones, Kerner, Schlenk, Schubert, Zirke '16

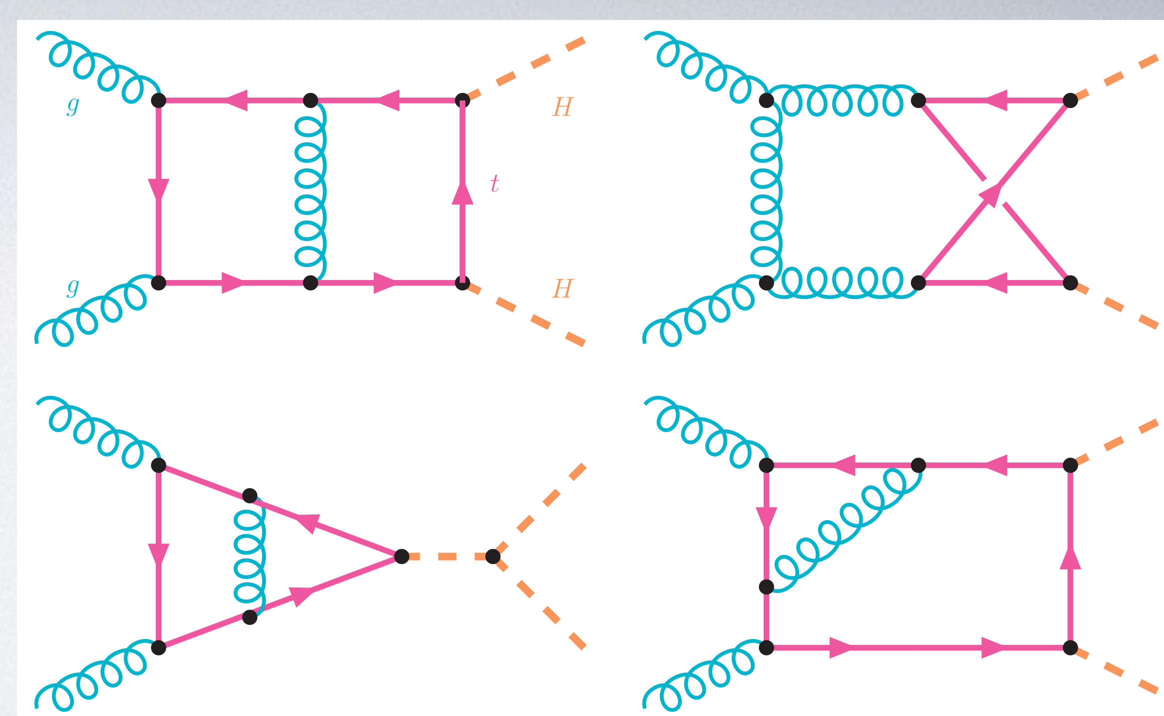
4 independent scales s12, s23, mH, mt

all integrals calculated **numerically** with

SecDec

Borowka, GH, Jones, Kerner, Schlenk, Zirke '15

Borowka, GH, Jahn, Jones, Kerner, Schlenk, Zirke '17



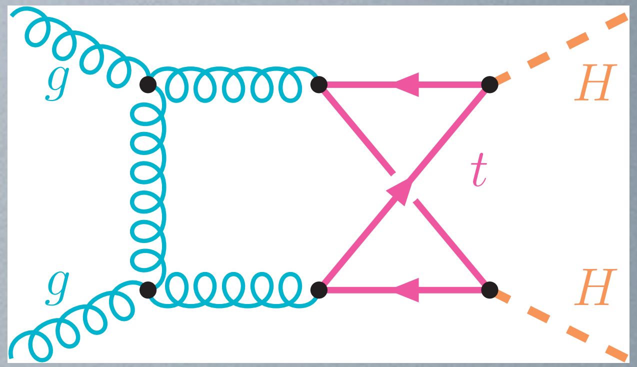
- q_T **resummation NLL+NLO**

Ferrera, Pires '16

MAX-PLANCK-GESELLSCHAFT

calculation: building blocks

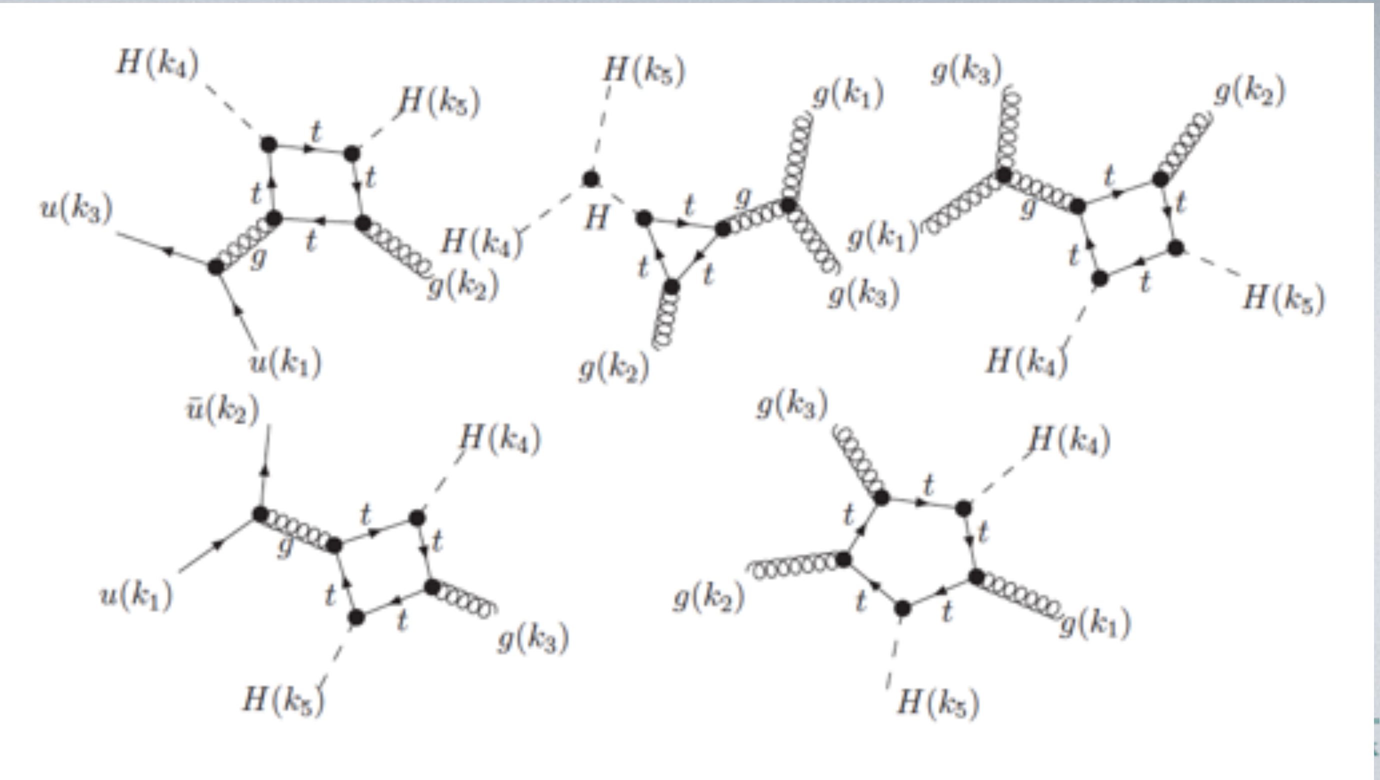
- amplitude generation with **GoSam-2loop** (python, QGRAF, FORM)
  [N.Greiner, S.Jahn, S.Jones, M.Kerner]
- amplitude reduction with **Reduze** [C. Studerus, A. v.Manteuffel]
- non-planar integrals computed mostly without reduction
- **all** integrals calculated numerically with **SecDec**
- total number of integrals:
  - before reduction: ~10000, after reduction ~330,
    after sector decomposition 11244 (3086 non-planar)
  - used finite basis for planar integrals



- real radiation:

  (a) GoSam-1L + Catani-Seymour dipole subtraction

  (b) GoSam-1L + POWHEG

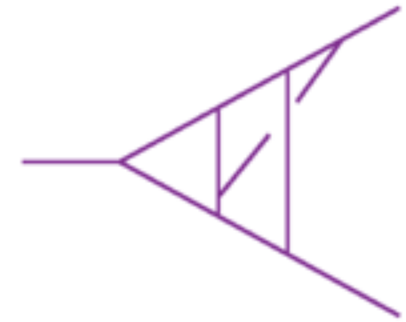


MAX-PLANCK-GESELLSCHAFT

numerical evaluation of multi-loop integrals (advert break)

http://secdec.hepforge.org

SecDec is hosted by Hepforge, IPPP Durham



SecDec

Sophia Borowka, Gudrun Heinrich, Stephan Jahn, Stephen Jones, Matthias Kerner, Johannes Schlenk, Tom Zirke

A program to evaluate dimensionally regulated parameter integrals numerically

home   download program   user manual   faq   changelog

NEW! Download the latest version of pySecDec as pySecDec-1.1.1.tar.gz. The manual is available here.

Download version 1.1 of pySecDec as pySecDec-1.1.tar.gz. The manual is available here.

The first release version of pySecDec can be downloaded as pySecDec-1.0.tar.gz. The manual is available here.
See also the corresponding paper arXiv:1703.09692.

Version 3.0 of the program can be downloaded as SecDec-3.0.9.tar.gz. The manual for this version is available here.

algorithm:	T. Binoth, GH ‘00
version 1.0:	J. Carter, GH ‘10
version 2.0:	S.Borowka, J. Carter, GH ‘12
version 3.0:	S.Borowka, GH, S.Jones, M.Kerner, J.Schlenk, T.Zirke ‘15
pySecDec:	S.Borowka, GH, S.Jahn, S.Jones, M.Kerner, J.Schlenk, T.Zirke ‘17

new

MAX-PLANCK-GESELLSCHAFT

Max-Planck-Institut für Physik (Werner-Heisenberg-Institut)

top mass effects

total cross sections at 14 TeV $\mu_0 = m_{HH}/2$

	$\sigma_{\rm LO}$[fb]	$\sigma_{\rm NLO}$[fb]	$\sigma_{\rm NNLO}$[fb]
HEFT	$17.07^{+30.9\%}_{-22.2\%}$	$31.93^{+17.6\%}_{-15.2\%}$	$37.52^{+5.2\%}_{-7.6\%}$
B-i. HEFT	$19.85^{+27.6\%}_{-20.5\%}$	$38.32^{+18.1\%}_{-14.9\%}$	
$\rm FT_{approx}$	$19.85^{+27.6\%}_{-20.5\%}$	$34.26^{+14.7\%}_{-13.2\%}$	
full m_t dep.	$19.85^{+27.6\%}_{-20.5\%}$	$32.91^{+13.6\%}_{-12.6\%}$	

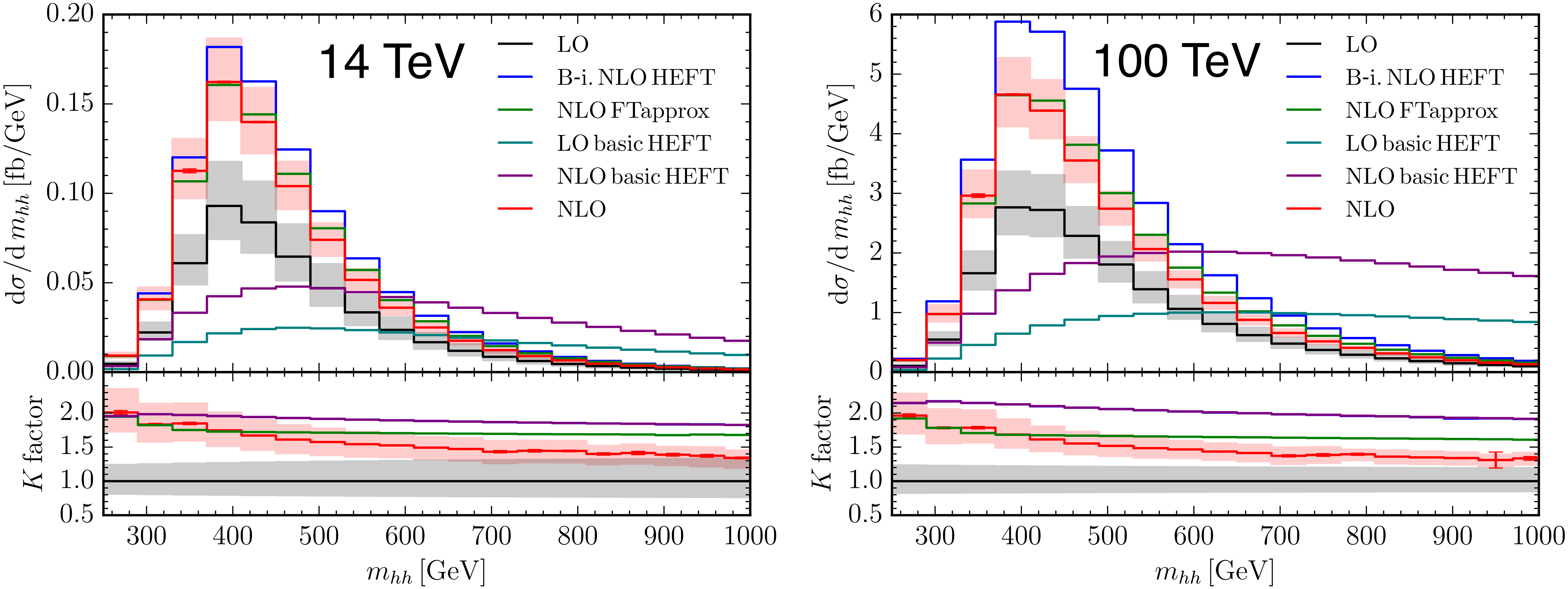
PDF4LHC15_nlo_30_pdfas

m_H=125 GeV, m_t=173 GeV

$$\sigma'_{NNLL} = \sigma_{NNLL} + \delta_t\,\sigma^{\rm HEFT}_{NLO} = 39.64^{+4.4\%}_{-6.0\%}$$

uncertainties: $\mu_{R,F} \in [\mu_0/2, 2\,\mu_0]$ (7-point variation)

Higgs boson pair invariant mass



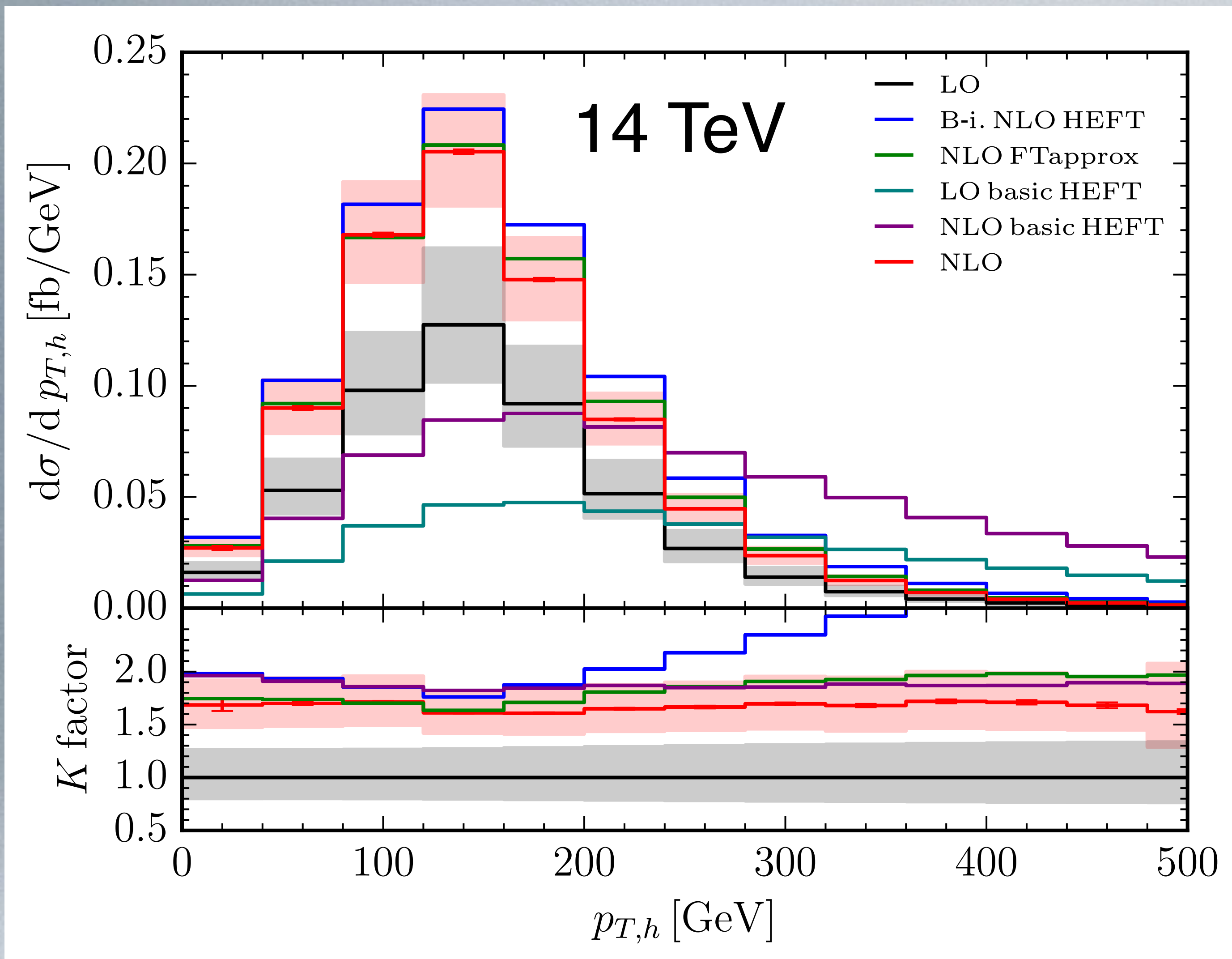
for large invariant masses:

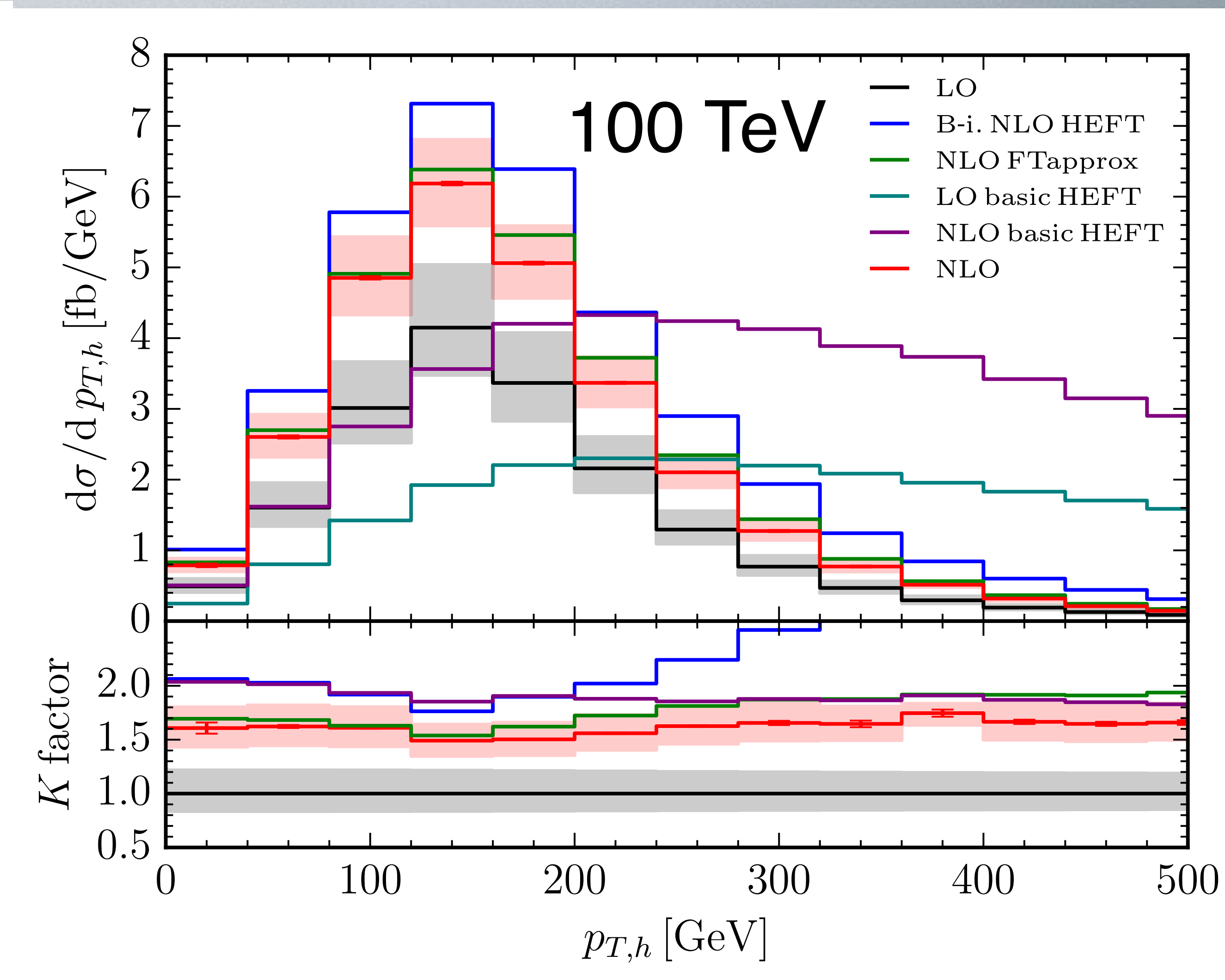
Born-improved NLO HEFT overestimates by about 50%, FTapprox by about 40% (at 14 TeV, worse at 100 TeV)

top quark loops resolved → HEFT has wrong scaling behaviour at high energies

top mass effects: II. distributions

transverse momentum of one of the Higgs bosons



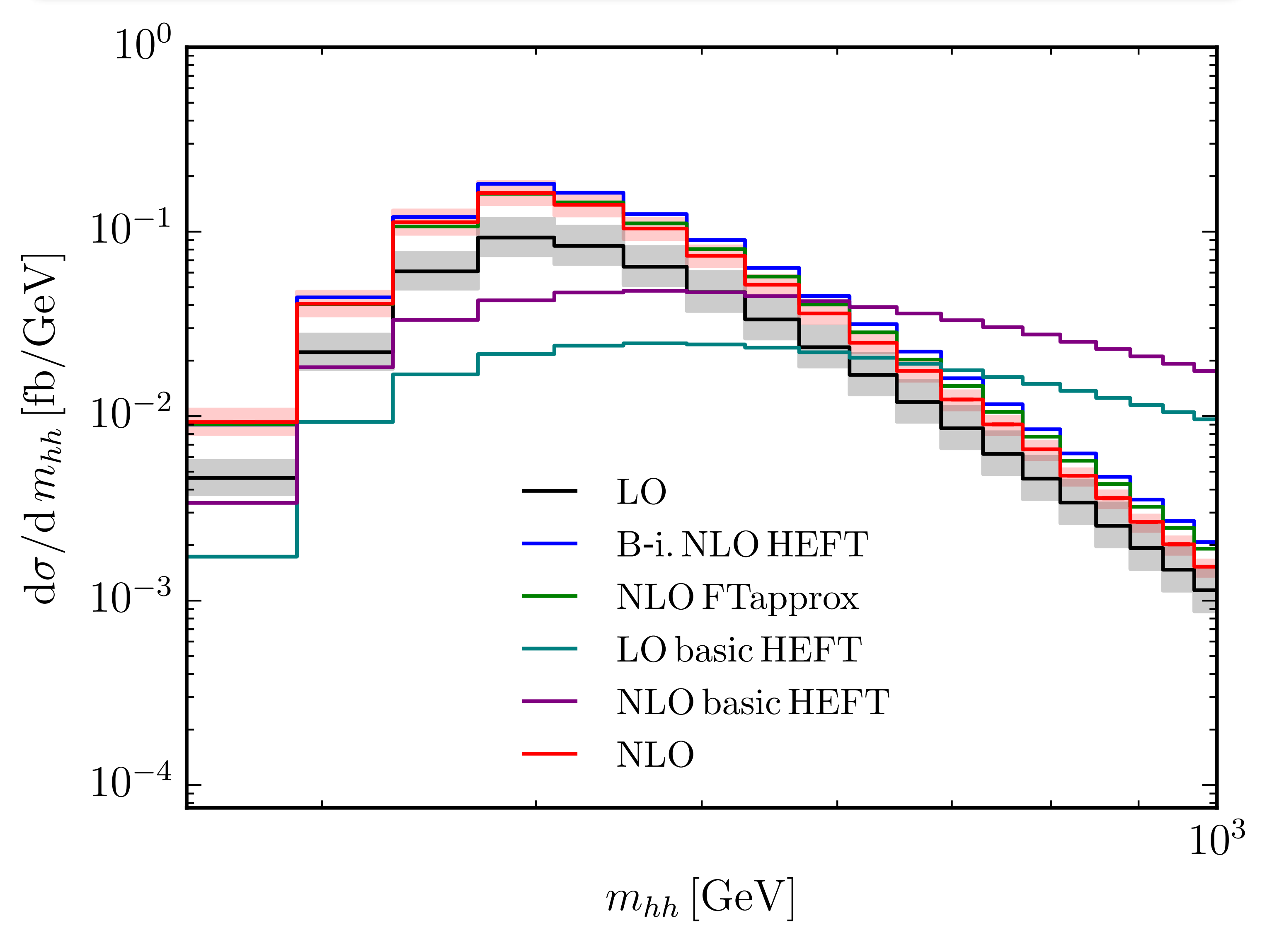


Born-improved NLO HEFT very poor at large pT

MAX-PLANCK-GESELLSCHAFT

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

scaling behaviour



$$\frac{d\hat{\sigma}}{dm_{hh}} \sim m_{hh}^{-3}$$ i.e. partonic cross section scales as $\hat{s}^{-1}$

HEFT approximation: $$\frac{d\hat{\sigma}}{dm_{hh}} \sim m_{hh}$$ i.e. $\hat{\sigma} \sim \hat{s}$

similar for H+jet: [Greiner, Höche, Luisoni, Schönherr, Winter '16]

$$\frac{d\hat{\sigma}}{dp_{T,h}} \sim 1/p_{T,h}^{a} \text{ with } a = 2(\text{full}), a = 1(\text{HEFT})$$

NLO-improved NNLO HEFT

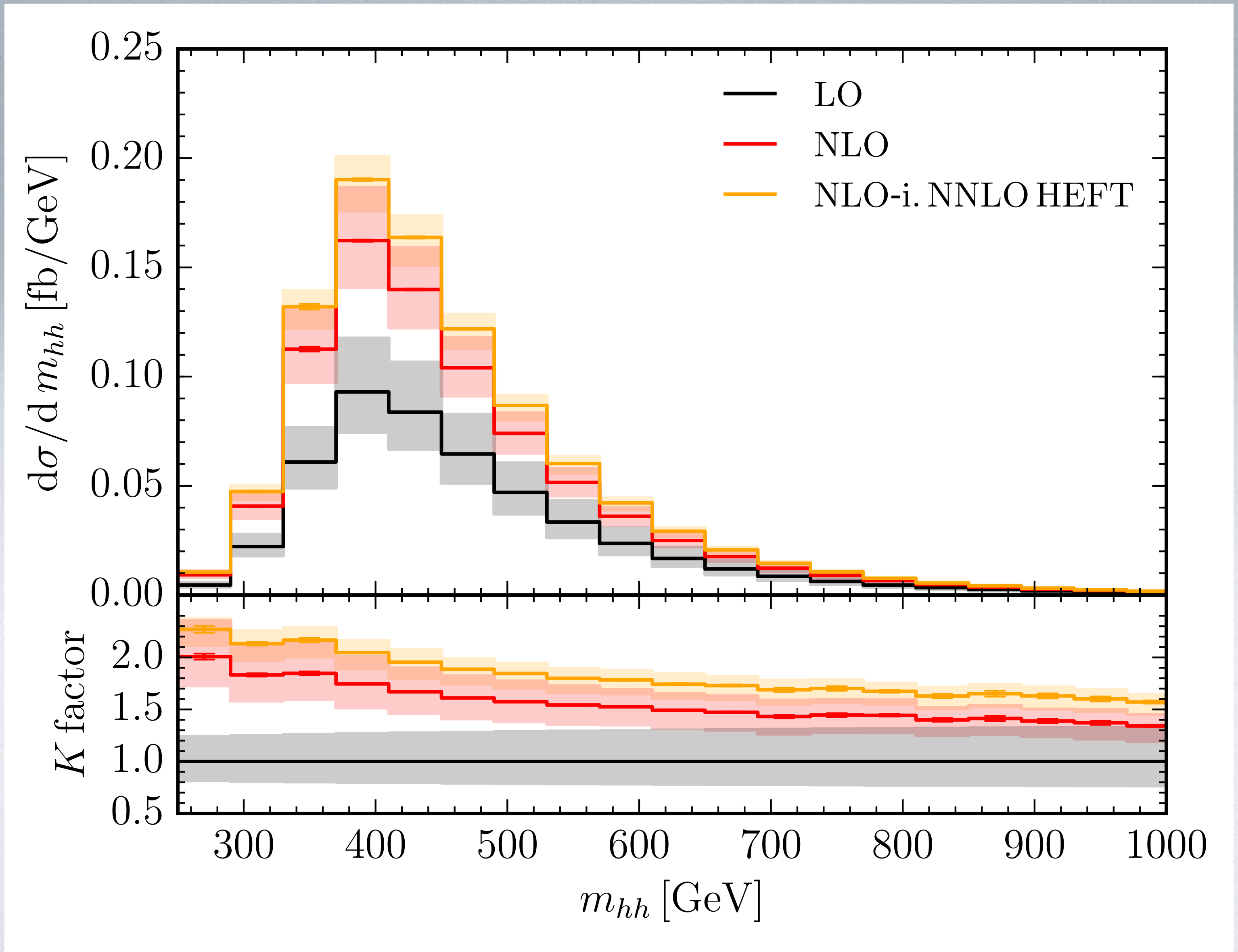
NNLO HEFT:

De Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev, arXiv:1606.09519

what we did in arXiv:1608.04798:

$$\frac{d\sigma^{\mathrm{NLO-i.NNLO\,HEFT}}}{dm_{hh}} = \frac{d\sigma_{\mathrm{NLO}}}{dm_{hh}} \times \frac{d\sigma^{\mathrm{HEFT}}_{\mathrm{NNLO}}/dm_{hh}}{d\sigma^{\mathrm{HEFT}}_{\mathrm{NLO}}/dm_{hh}}$$

bin-by-bin rescaling at observable level by NNLO HEFT K-factor



combination with parton showers

GH, S.Jones, M.Kerner, G.Luisoni, E.Vryonidou 1703.09252

- avoid evaluation of two-loop amplitude for each phase space point
- two-loop amplitude depends only on $\hat{s}, \hat{t} \quad (m_t, m_H \text{ fixed})$

→ construct 2-dim grid

- variable transformation to achieve more uniform distribution

$$x = f(\beta(\hat{s})), \quad c_\theta = |\cos\theta| = \left|\frac{\hat{s} + 2\hat{t} - 2m_H^2}{\hat{s}\beta(\hat{s})}\right| \qquad \beta(\hat{s}) = \sqrt{1 - 4m_H^2/\hat{s}}$$

- choose f according to cumulative distribution of phase space points
- use *SciPy* package for interpolation [Clough, Tocher]

combination with parton showers

combination with both POWHEG and MadGraph5_aMC@NLO

- different matching schemes
- same shower (Pythia 8.2)
- no Higgs decays, no hadronisation

combination with parton showers

combination with both POWHEG and MadGraph5_aMC@NLO

- different matching schemes
- same shower (Pythia 8.2)
- no Higgs decays, no hadronisation

POWHEG User-Process-V2/ggHH

2-loop results publicly available, easy to use!

MAX-PLANCK-GESELLSCHAFT

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

combination with parton showers

combination with both POWHEG and MadGraph5_aMC@NLO

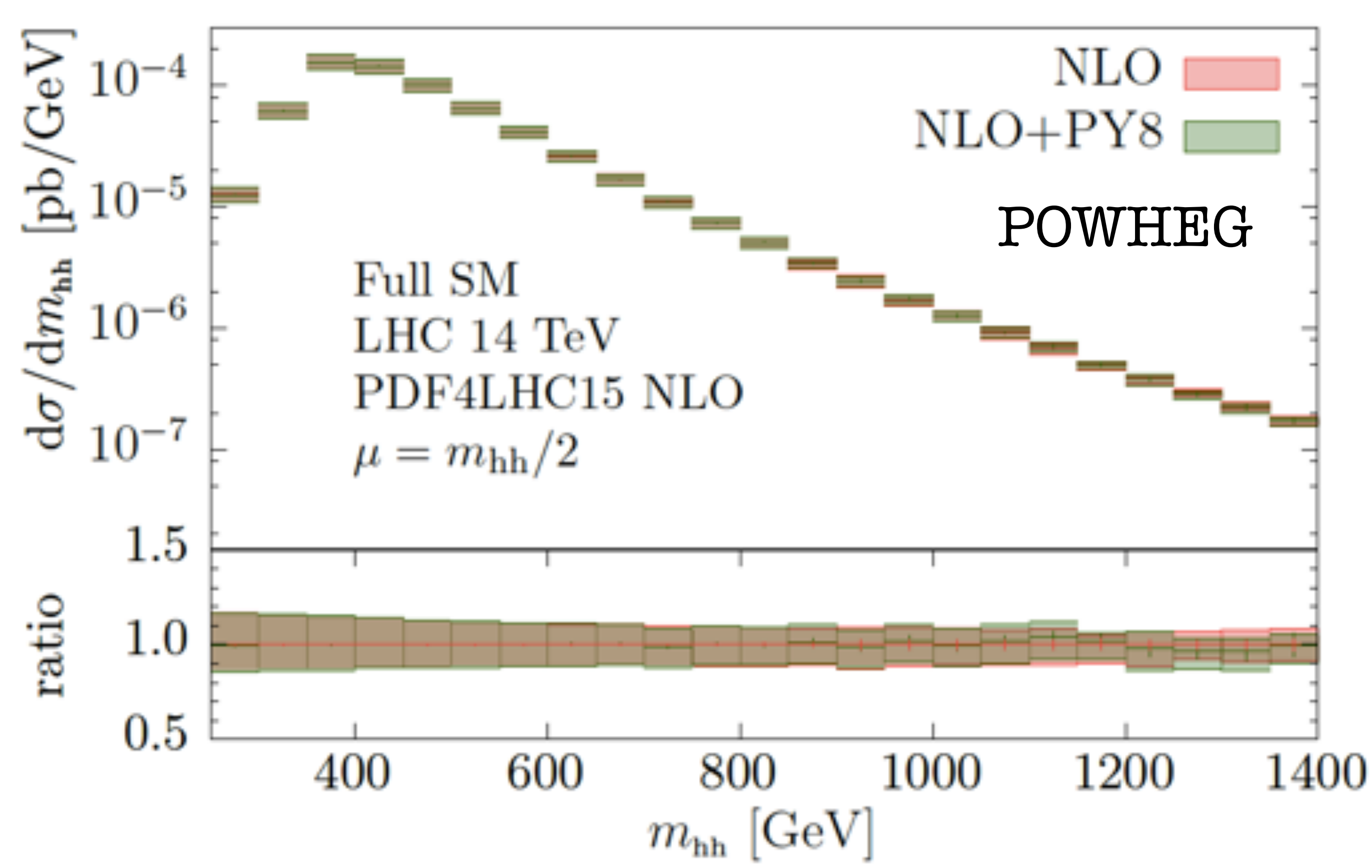
- different matching schemes
- same shower (Pythia 8.2)
- no Higgs decays, no hadronisation

POWHEG User-Process-V2/ggHH

2-loop results publicly available, easy to use!

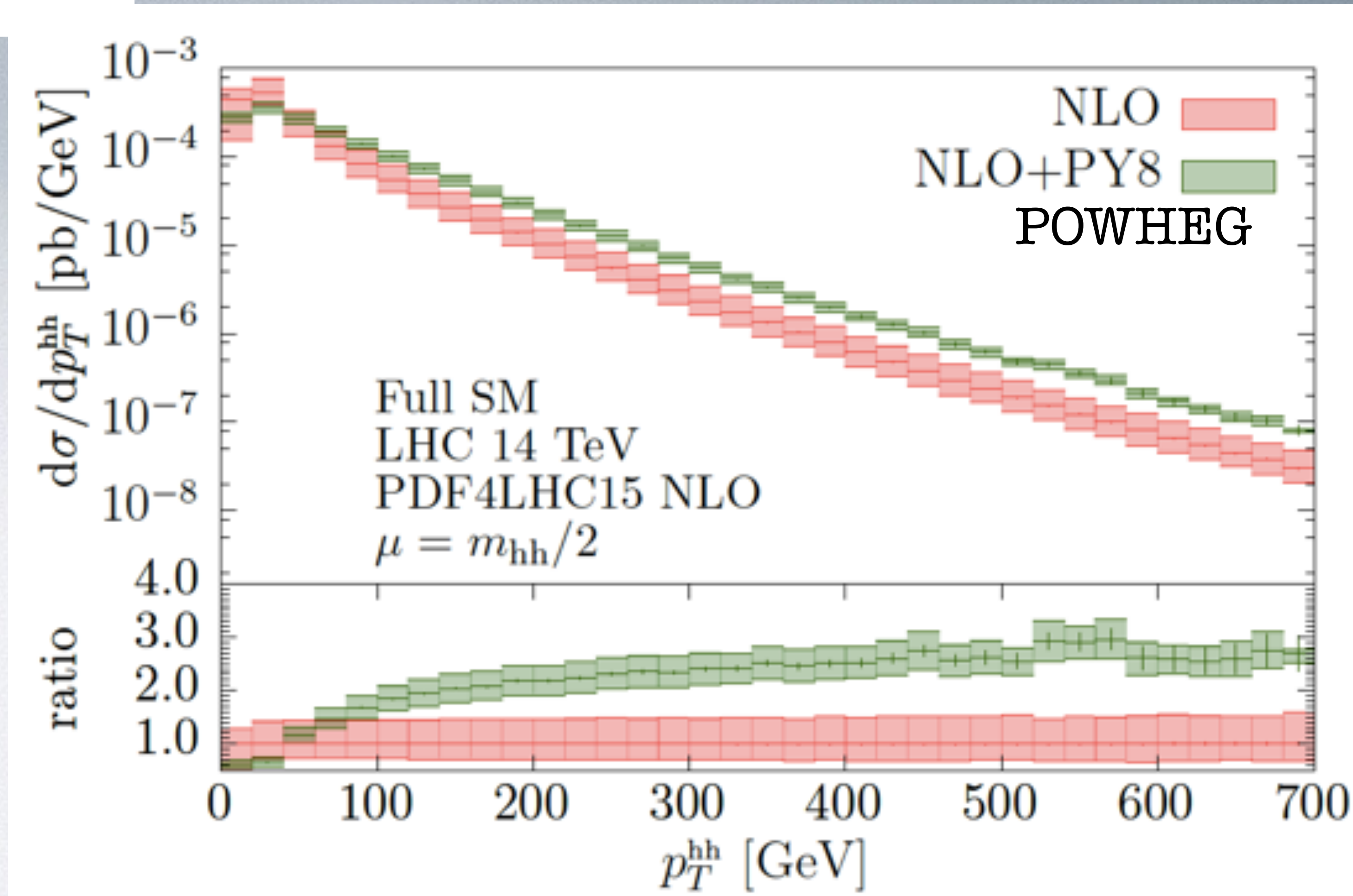
combination with Herwig7.1 and Sherpa is on the way

compare fixed order and showered results



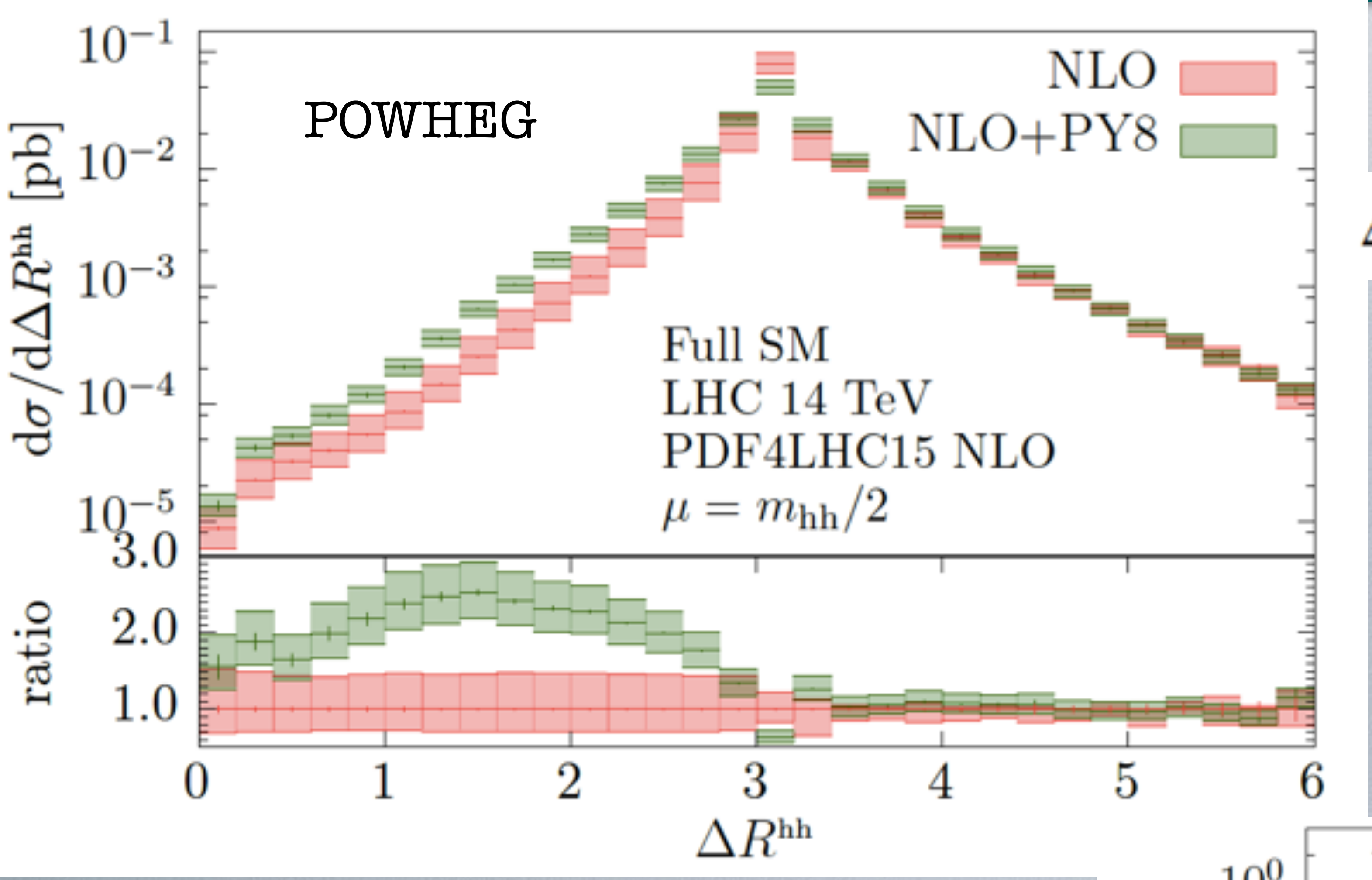
large shower effects on p_T^{hh}

expected because fixed order is first non-trivial order



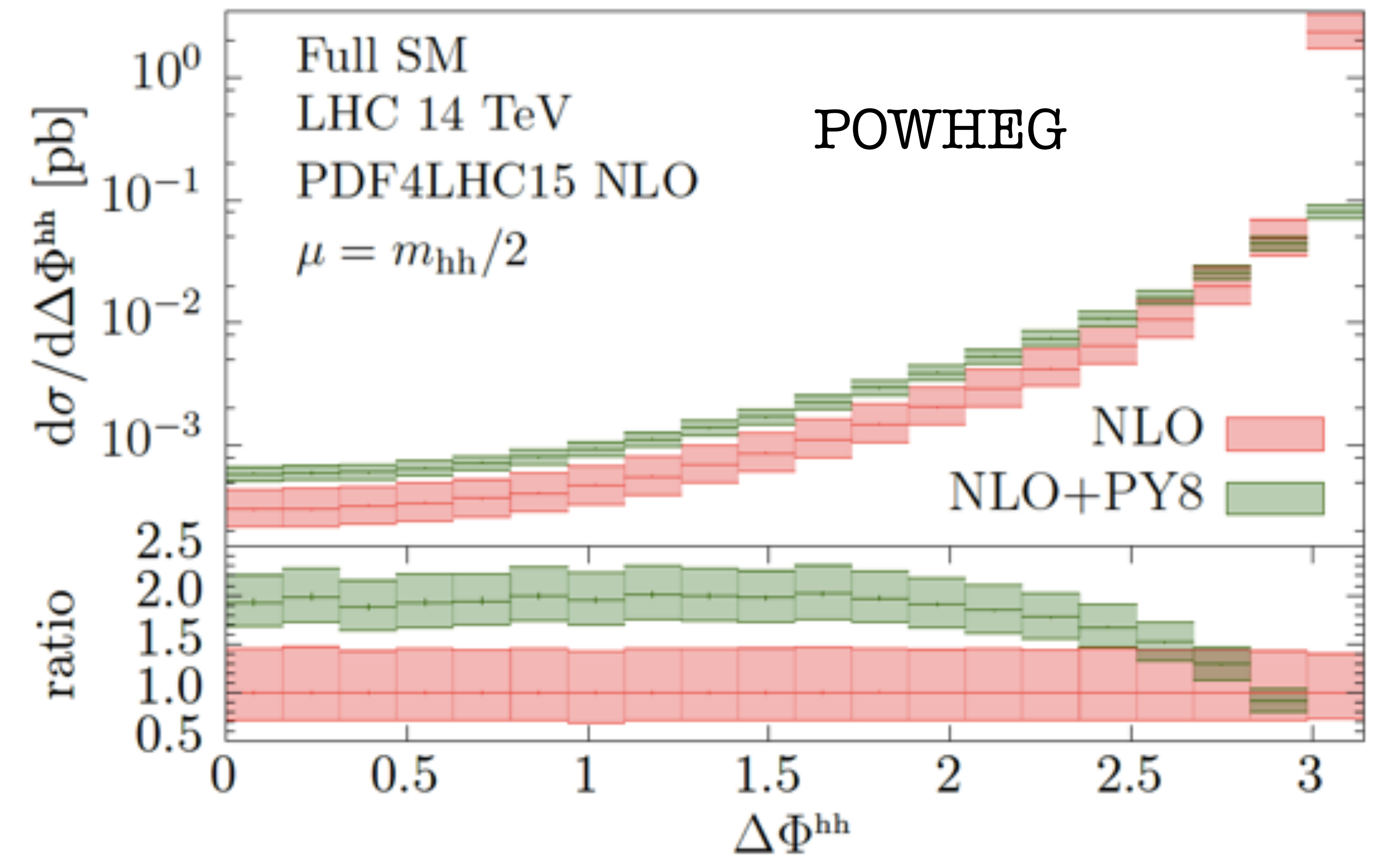
MAX-PLANCK-GESELLSCHAFT

compare fixed order and showered results



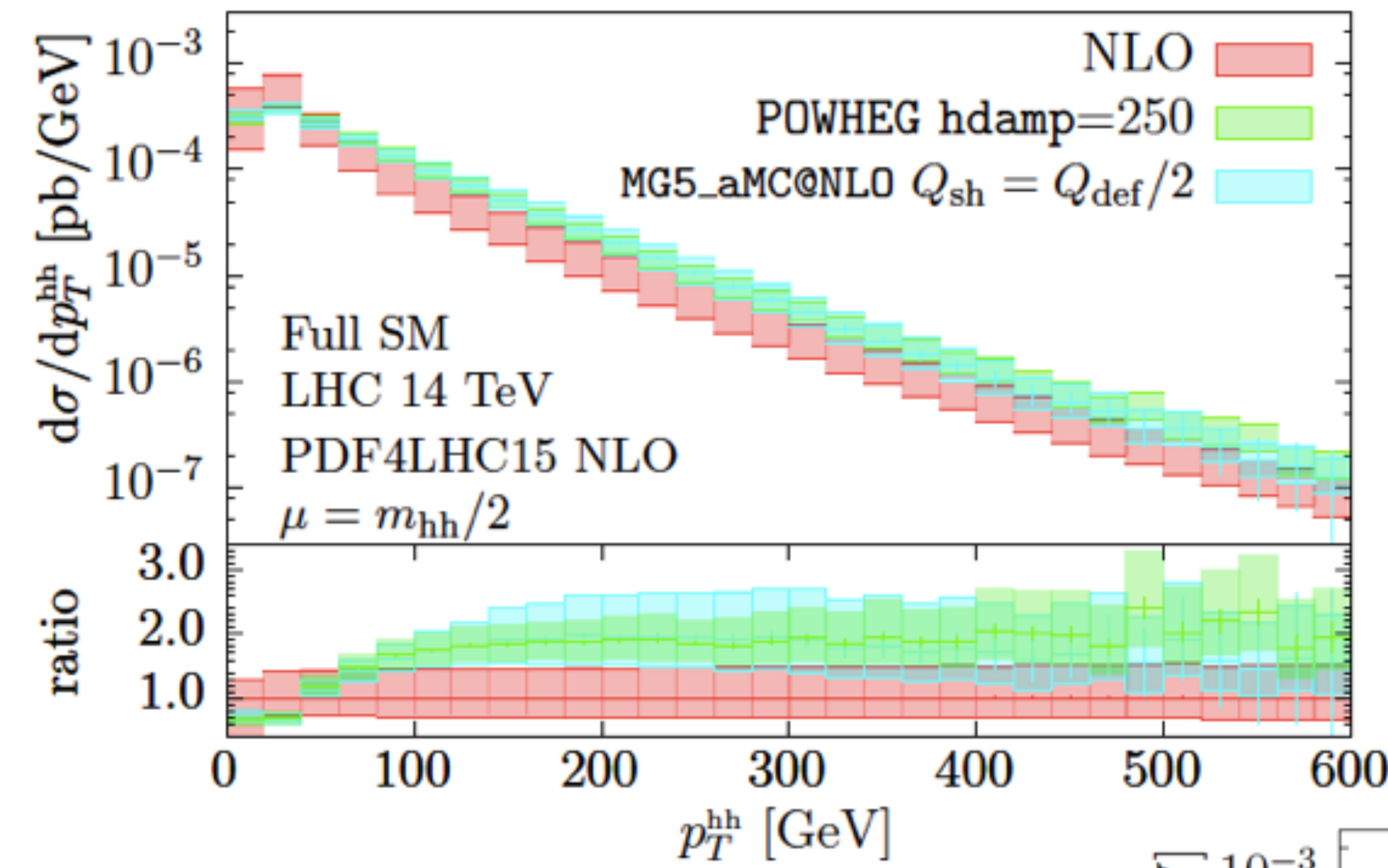
$$\Delta R^{hh} = \sqrt{(\eta_1 - \eta_2)^2 + (\Phi_1 - \Phi_2)^2}$$

for $\Delta R^{hh} < \pi$ fixed order is only "LO accurate"



MAX-PLANCK-GESELLSCHAFT

dependence on shower starting scale

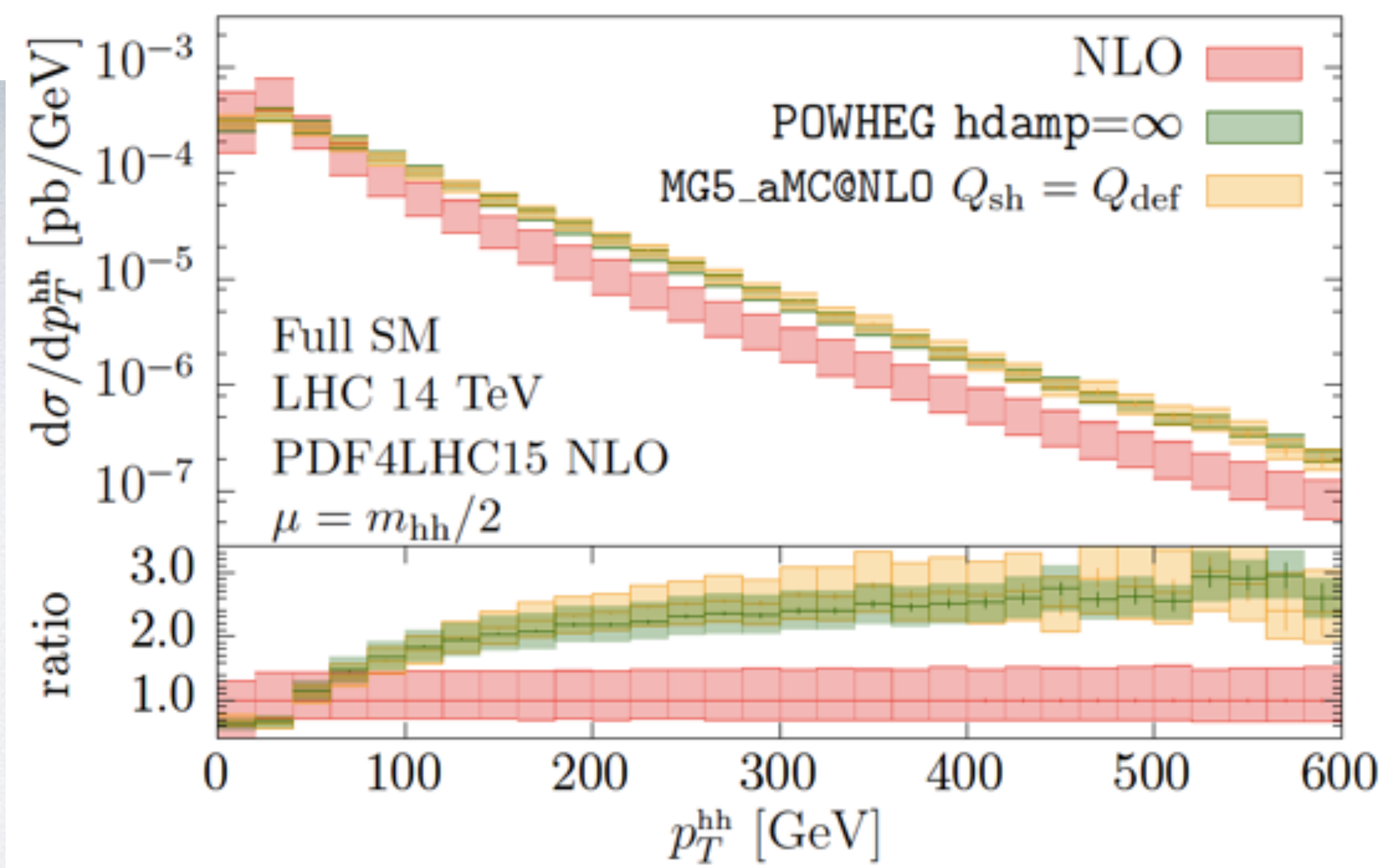


hdamp limits amount of exponentiated hard radiation

$$R_{\rm sing} = R \times F\,,$$

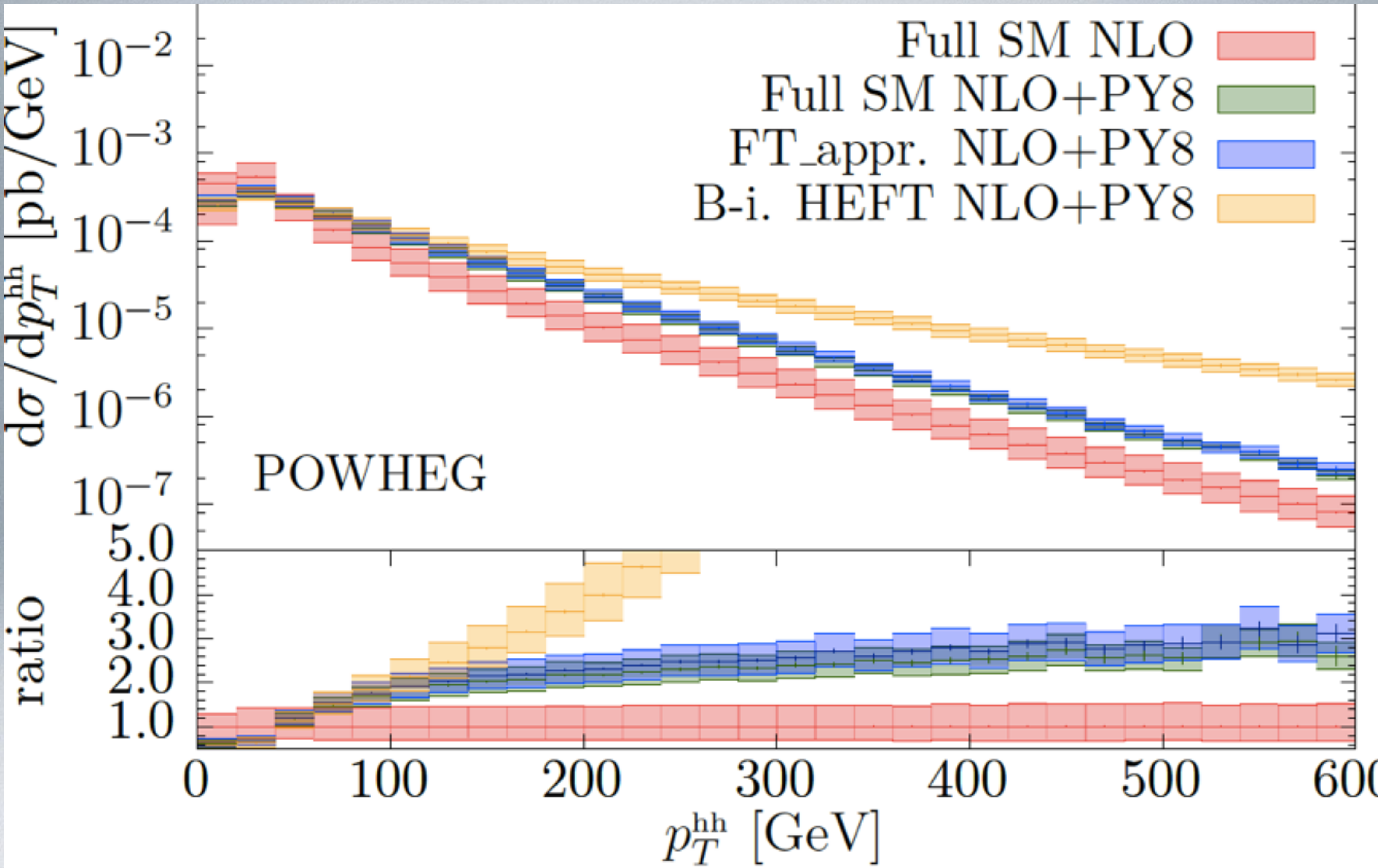
$$R_{\rm reg} = R \times (1 - F)$$

$$F = \frac{h^2}{(p_T^{\rm hh})^2 + h^2}$$



MAX-PLANCK-GESELLSCHAFT

compare different approximations



shower effects large but order(s) of magnitude smaller than difference to Born-improved HEFT

Summary

- Born-improved HEFT approximation fails to describe tails of distributions

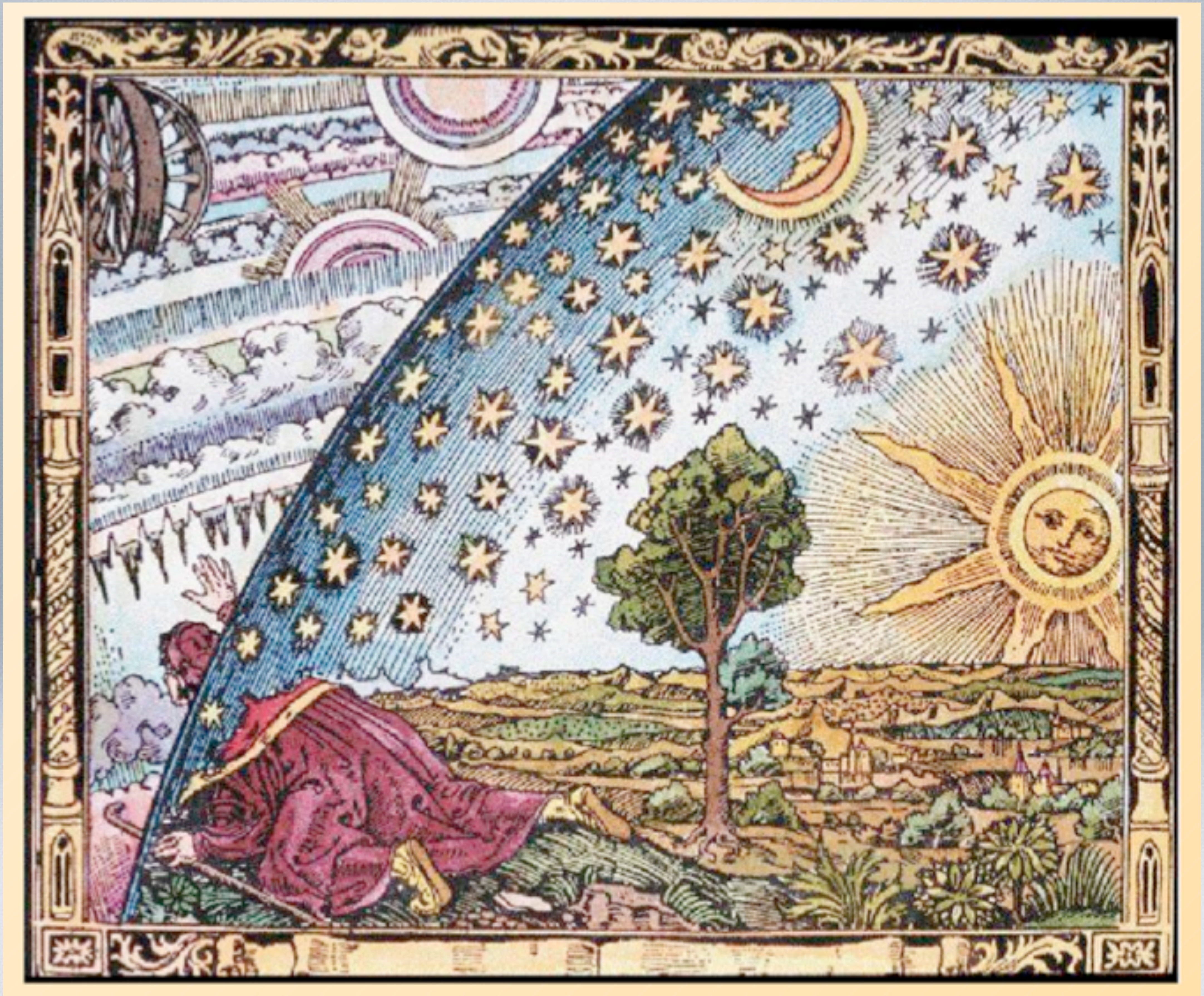
- FTapprox does a decent job for distributions/regions dominated by real radiation

- mass effects more important than shower effects

- **numerical** methods for 2-loop integrals can prove very useful in cases where analytic results are not available

MAX-PLANCK-GESELLSCHAFT

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

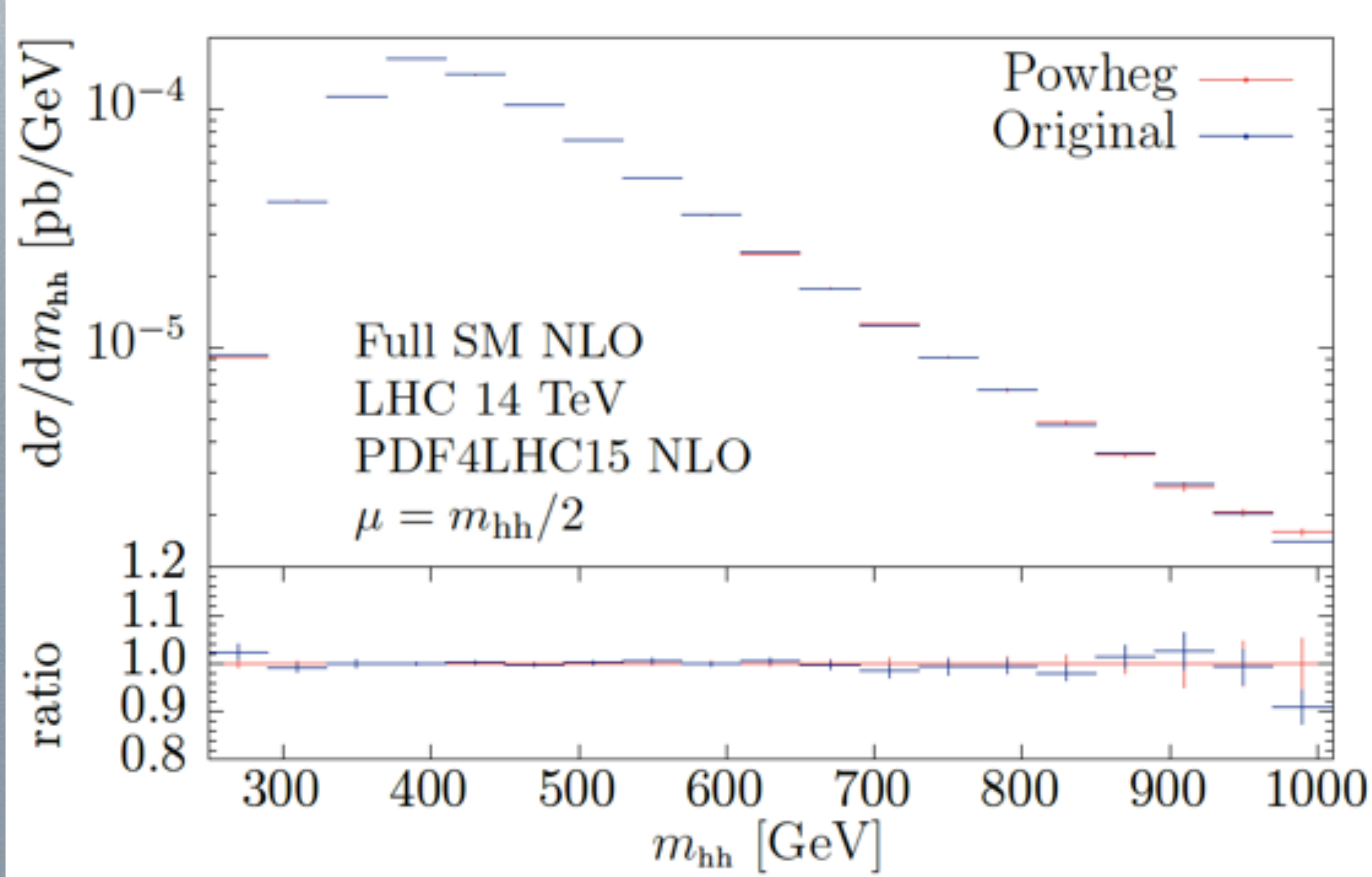


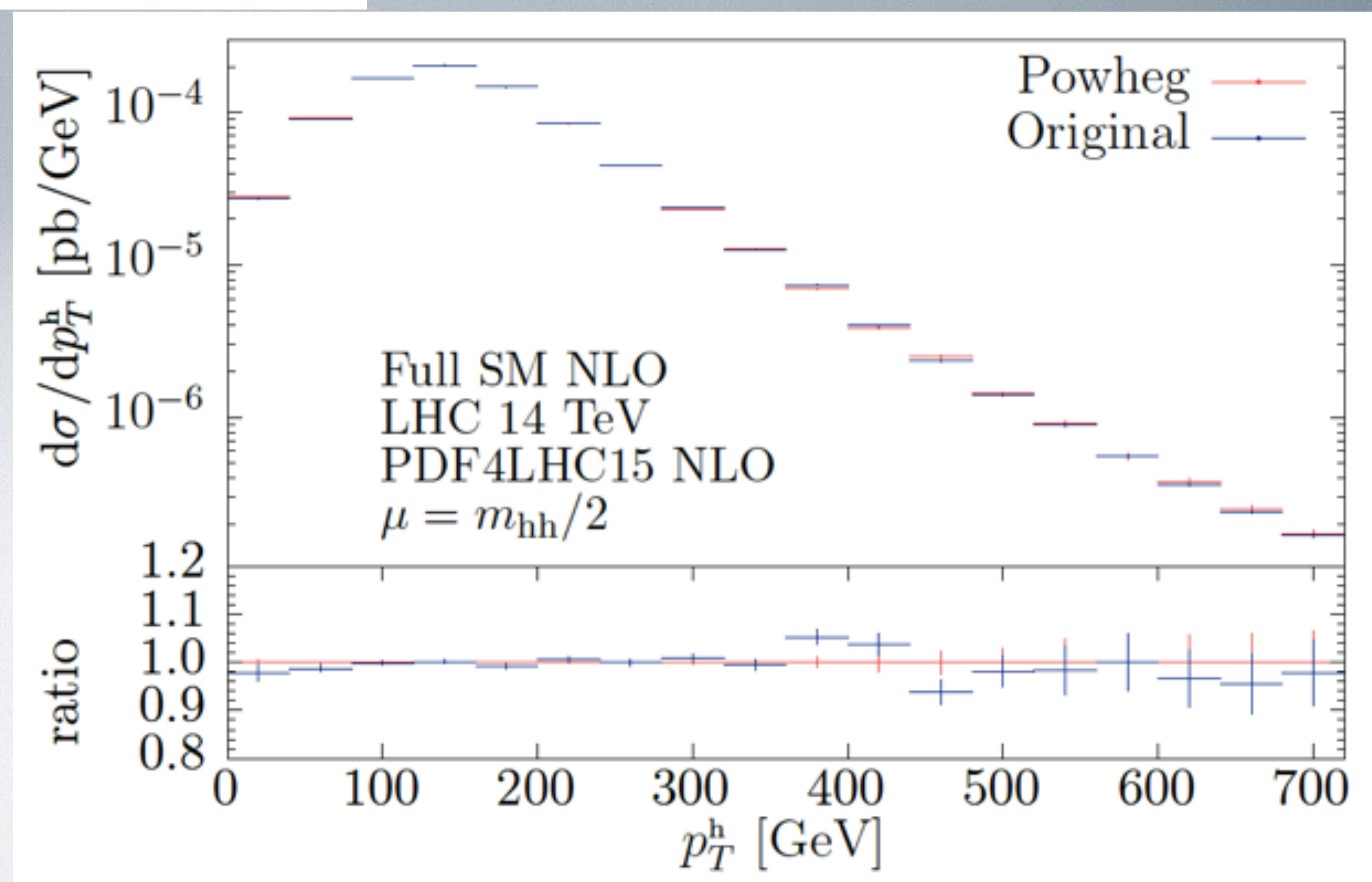
BACKUP SLIDES

MAX-PLANCK-GESELLSCHAFT

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

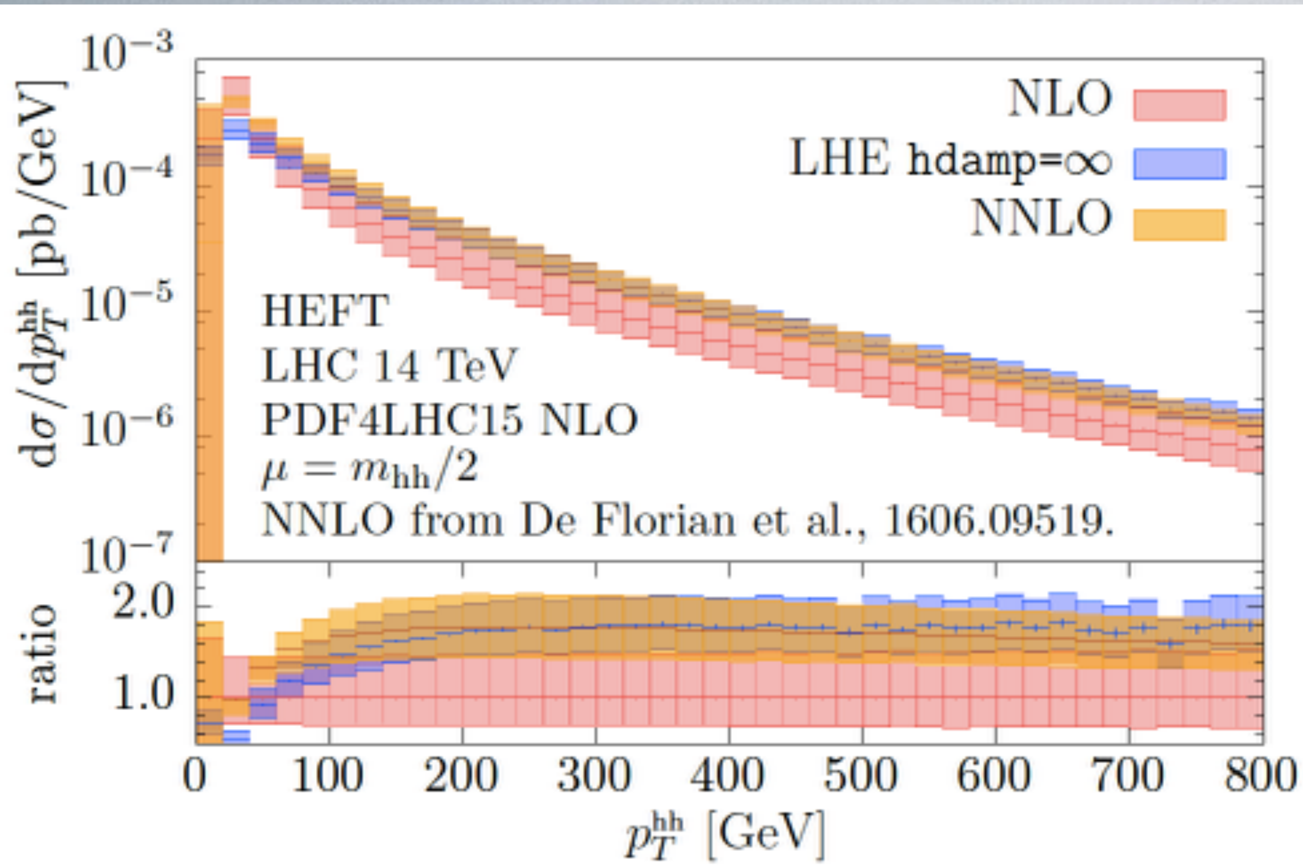
grid validation





MAX-PLANCK-GESELLSCHAFT

“NNLO” effects (HEFT)



hdamp limits amount of exponentiated hard radiation

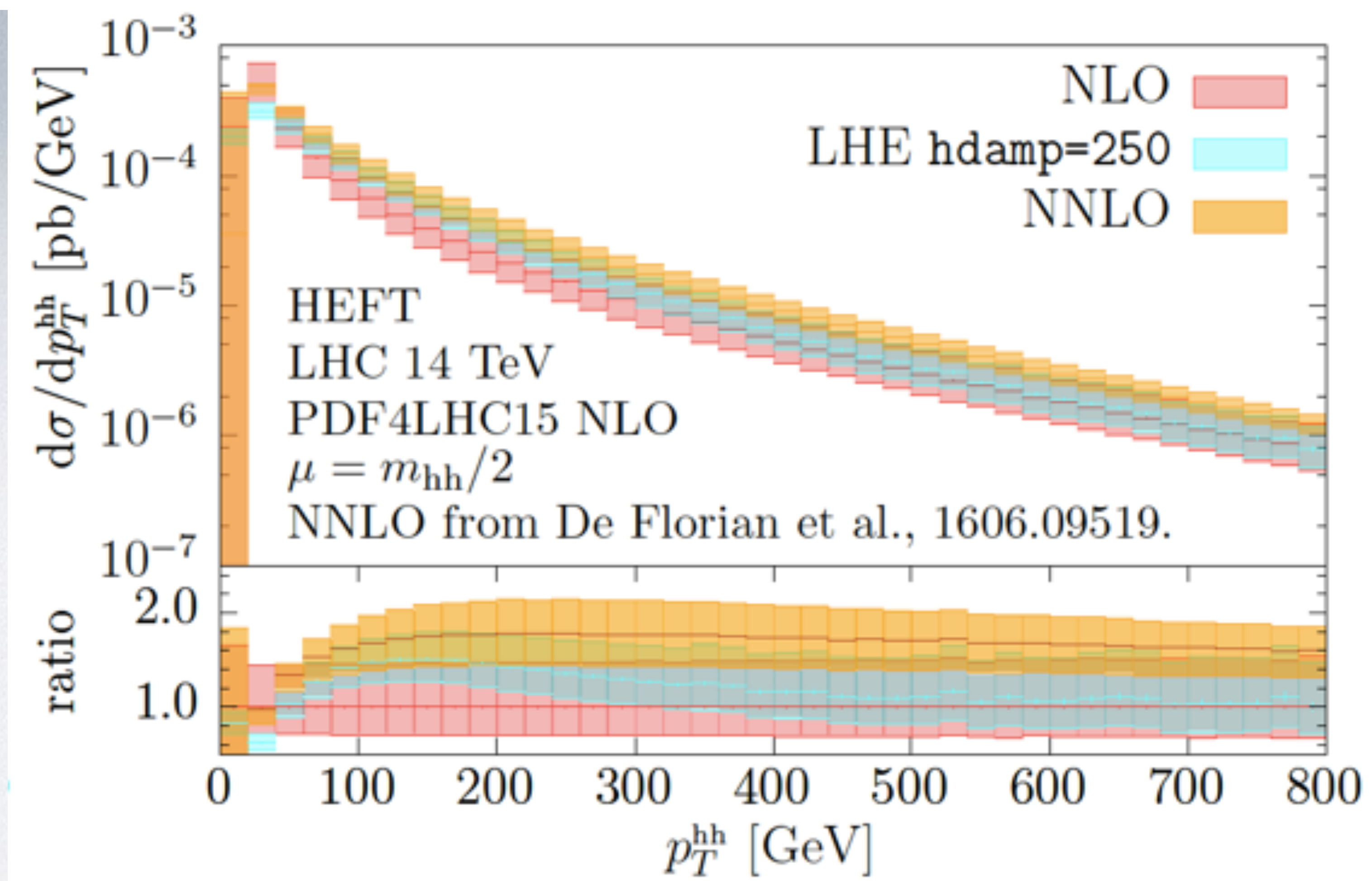
$$R_{\text{sing}} = R \times F\,,$$
$$R_{\text{reg}} = R \times (1 - F)$$

$$F = \frac{h^2}{(p_T^{hh})^2 + h^2}$$

basic HEFT approximation

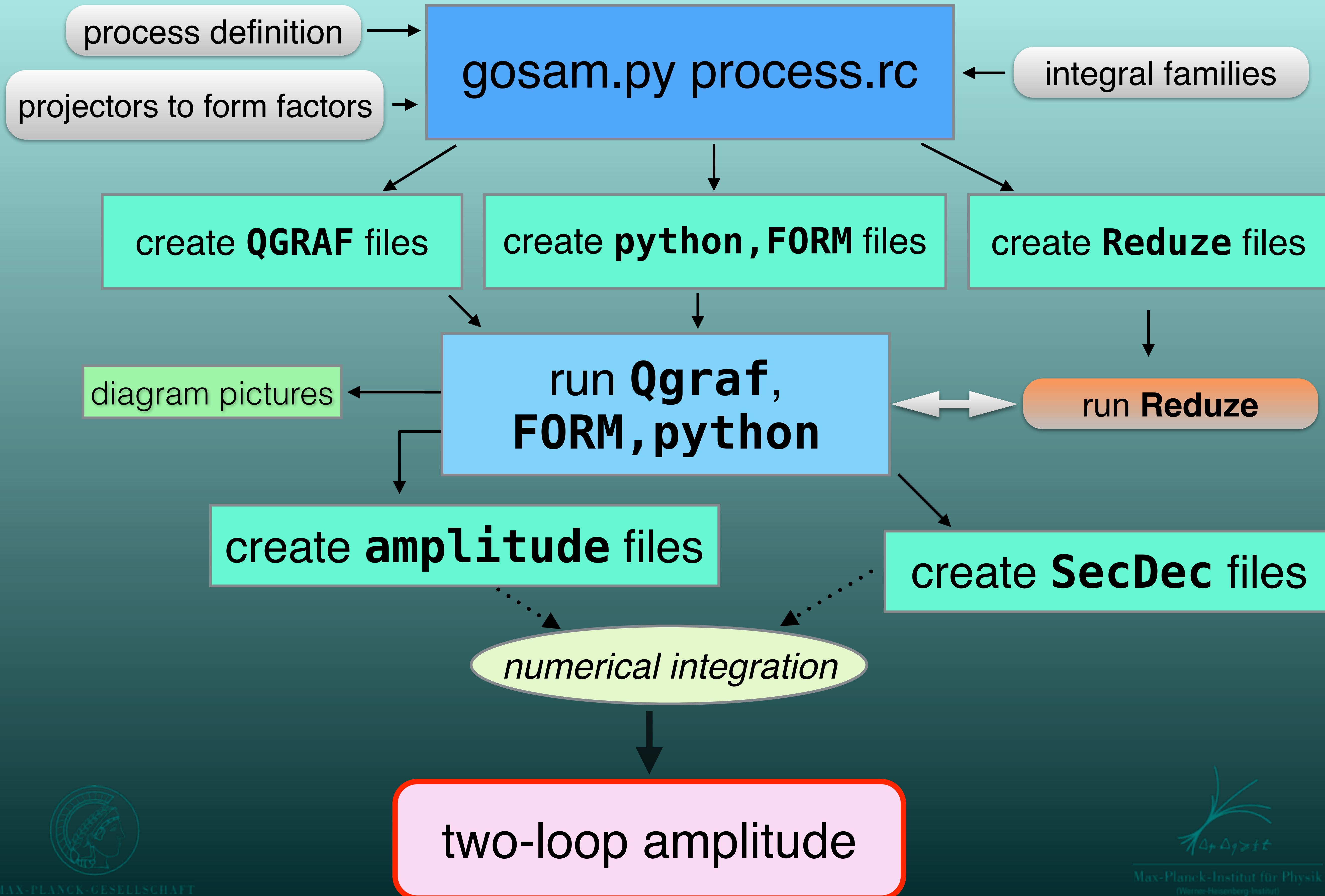
LHE: Les Houches event level

default hdamp=∞ close to NNLO in the tail



MAX-PLANCK-GESELLSCHAFT

automated 2-loop amplitudes: GoSam @ 2 loops



credits

GoSam 2-loop N.Greiner, GH, S.Jahn, S.Jones, M.Kerner, J.Schlenk et al.

QGRAF P. Nogueira

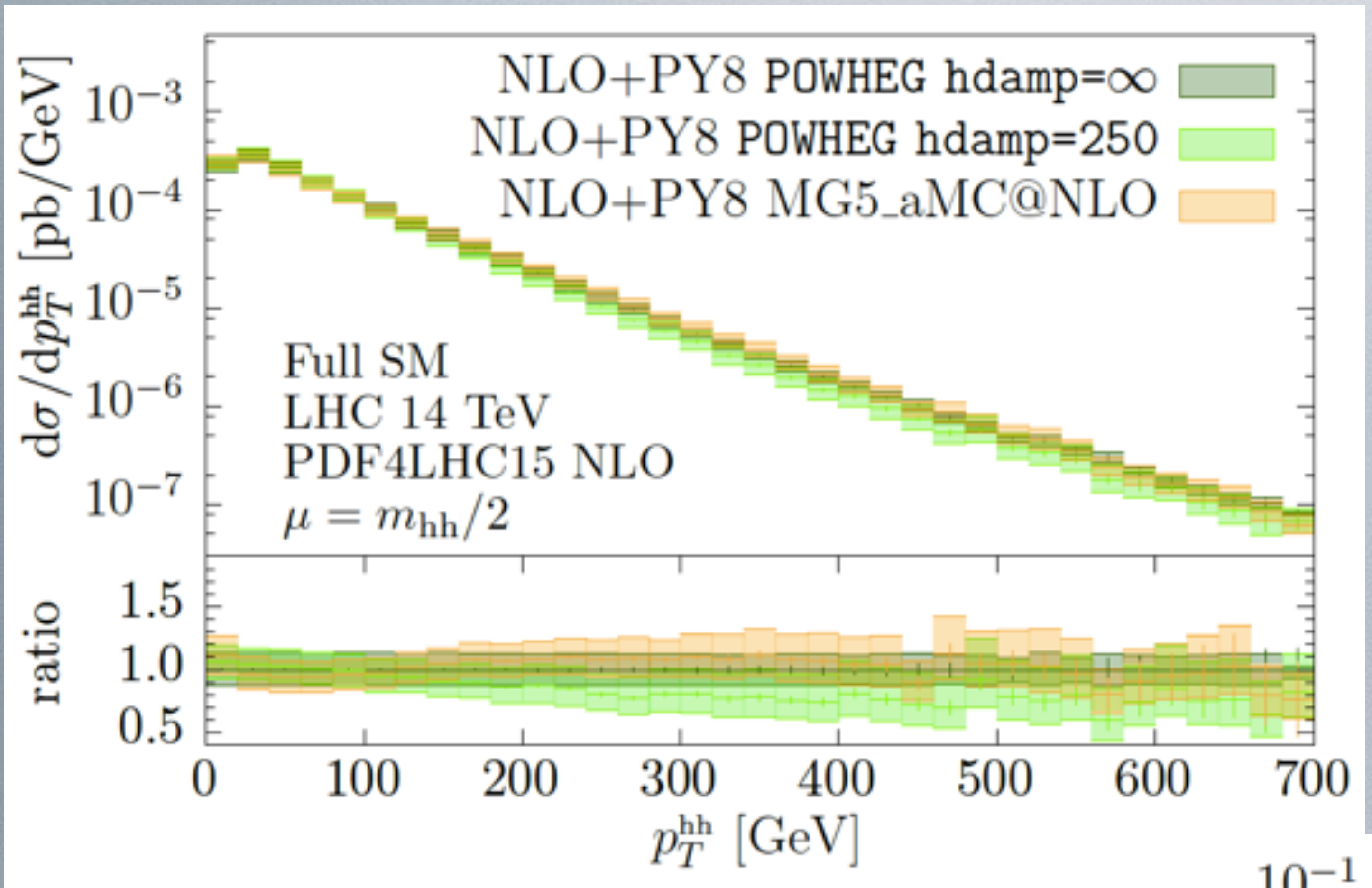
FORM J. Vermaseren, J. Kuipers, B. Ruijl, T. Ueda, J. Vollinga

Reduze C. Studerus, A. von Manteuffel

GoSam 1-loop T. Binoth, G.Cullen, H.van Deurzen, N.Greiner, GH, S.Jahn, G.Luisoni, P. Mastrolia, E.Mirabella, G. Ossola, T. Peraro, T. Reiter, J. Reichel, J. Schlenk, J.F. von Soden-Fraunhofen, F. Tramontano

SecDec S.Borowka, GH, S.Jahn, S.Jones, M.Kerner, J.Schlenk, T.Zirke

compare POWHEG and MG5_aMC@NLO



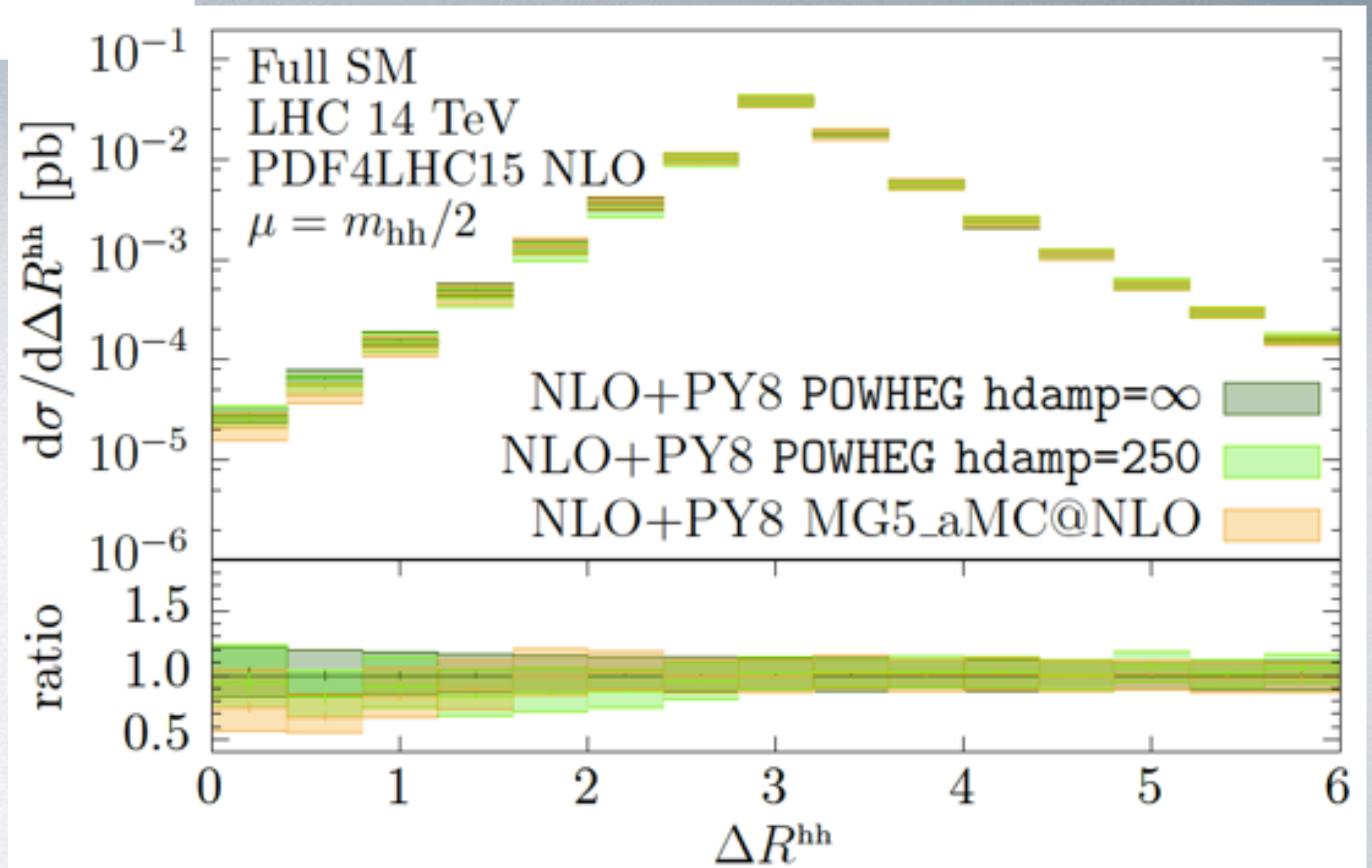
$$R_{\text{sing}} = R \times F\,,$$
$$R_{\text{reg}} = R \times (1 - F)$$

$$F = \frac{h^2}{(p_T^{hh})^2 + h^2}$$

h=hdamp limits amount of exponentiated hard radiation

default hdamp=∞

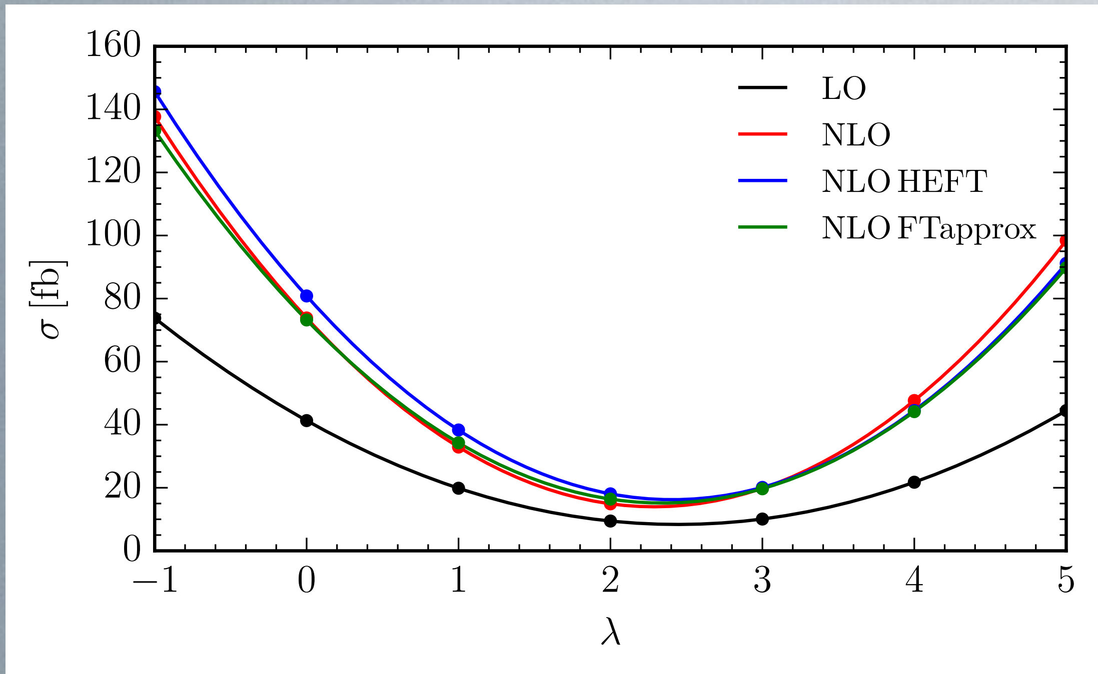
matching uncertainties small



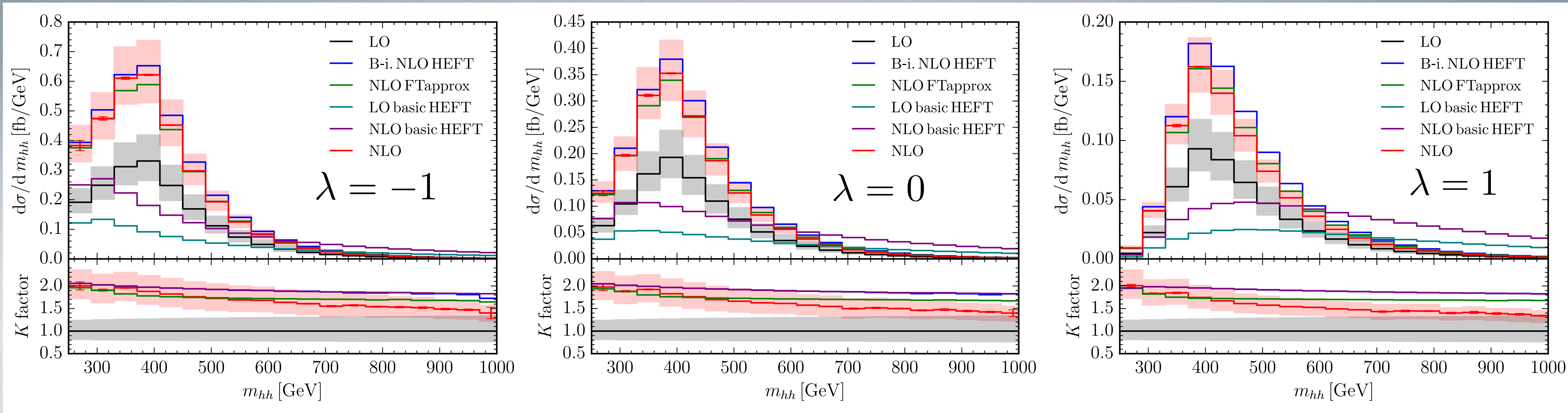
MAX-PLANCK-GESELLSCHAFT

variation of triple Higgs coupling

$$\lambda = \lambda_{BSM}/\lambda_{SM}$$



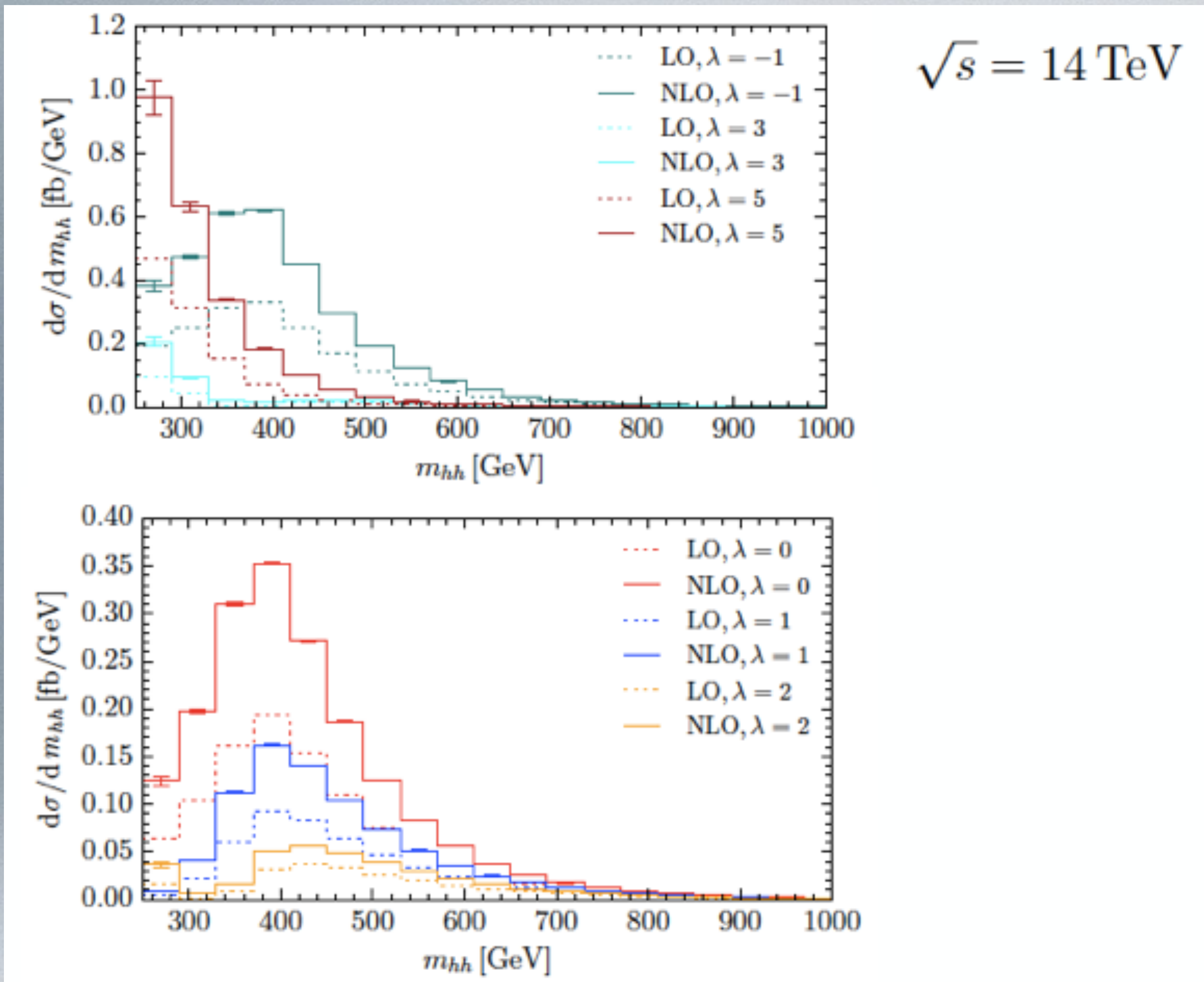
cross section has a minimum around $\lambda = 2$ due to destructive interference between diagrams containing λ and box-type diagrams

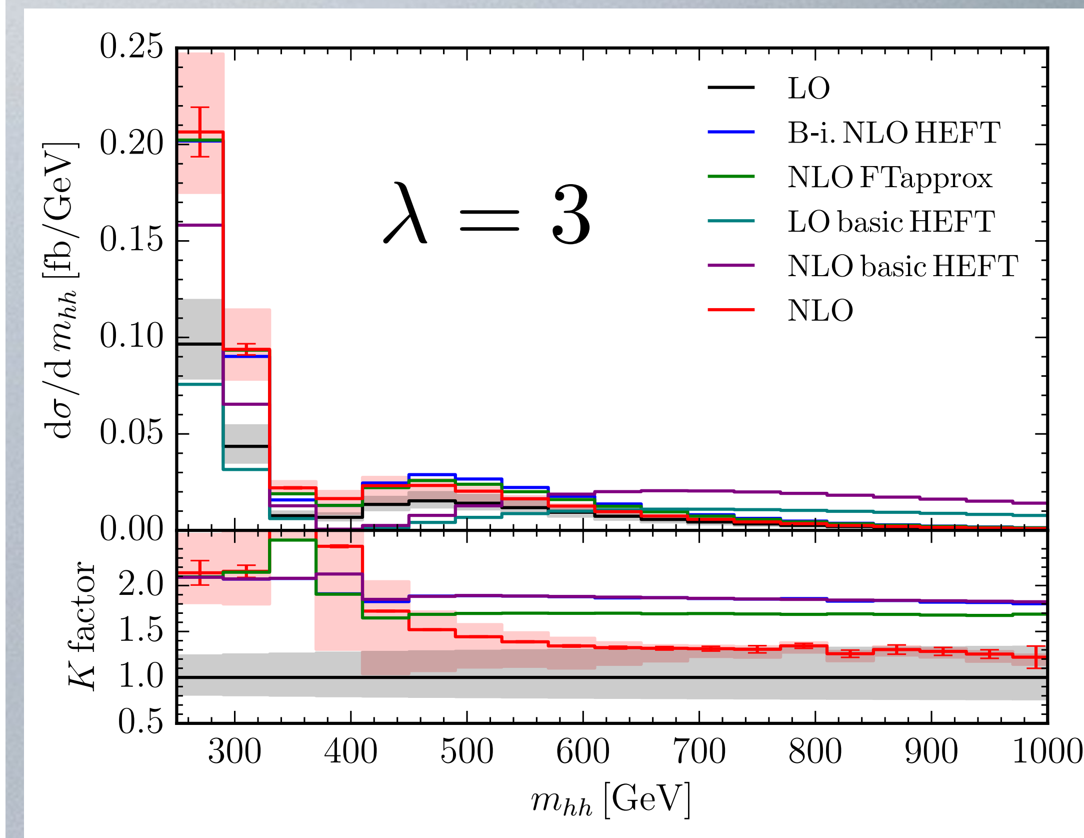


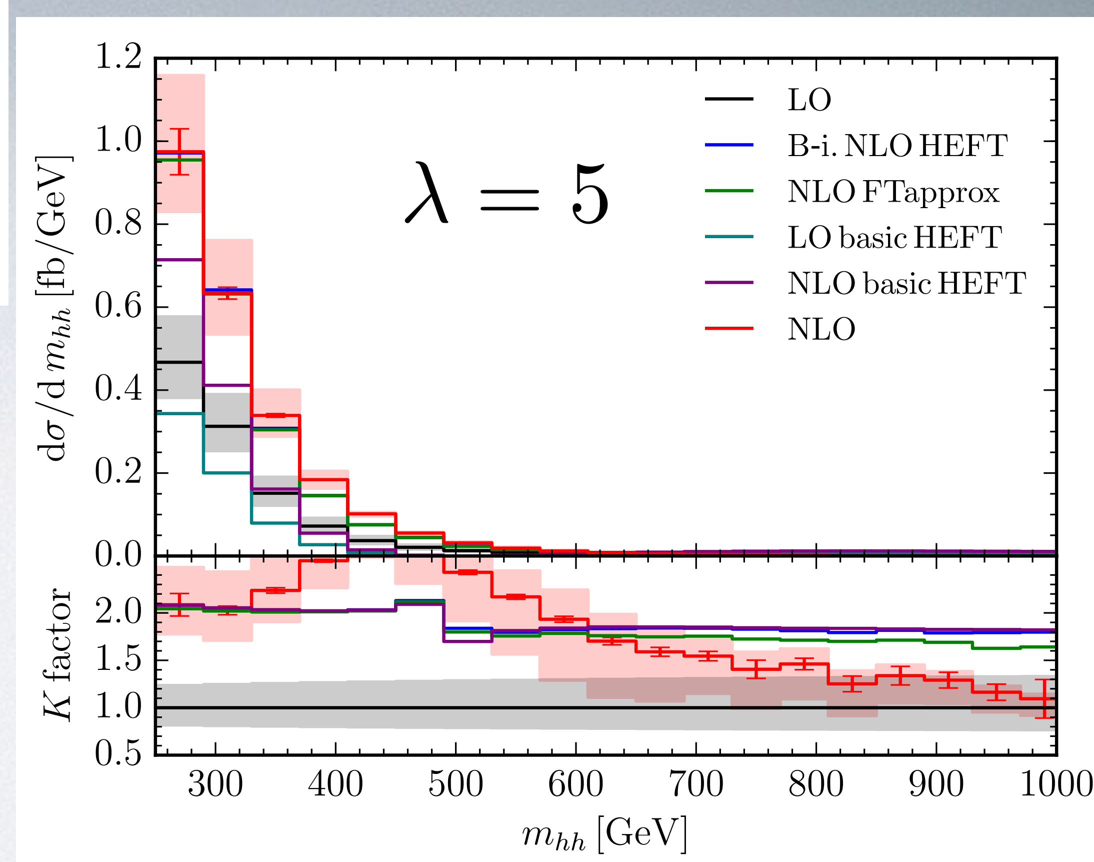
MAX-PLANCK-GESELLSCHAFT

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

variation of triple Higgs coupling







distributions have discriminating power

full analysis requires inclusion of other operators, e.g. $t\bar{t}hh$ coupling

MAX-PLANCK-GESELLSCHAFT

form factor decomposition

Expose tensor structure: $\mathcal{M} = \epsilon^1_\mu \epsilon^2_\nu \mathcal{M}^{\mu\nu}$

Form Factors (Contain integrals)

$$\mathcal{M}^{\mu\nu} = F_1(\hat{s}, \hat{t}, m_h^2, m_t^2, D) T_1^{\mu\nu} + F_2(\hat{s}, \hat{t}, m_h^2, m_t^2, D) T_2^{\mu\nu}$$

(Tensor) Basis, built from external momenta & metric

Choose: $\mathcal{M}^{++} = \mathcal{M}^{--} = -F_1$

$\mathcal{M}^{+-} = \mathcal{M}^{-+} = -F_2$

Self-coupling diagrams are 1PR by cutting a scalar propagator

By angular momentum conservation they contribute only to F_1

$$T_1^{\mu\nu} = g^{\mu\nu} - \frac{p_2^\mu p_1^\nu}{p_1 \cdot p_2} \qquad p_T^2 = \frac{ut - m_H^4}{s}$$

$$T_2^{\mu\nu} = g^{\mu\nu} + \frac{m_H^2 p_2^\mu p_1^\nu}{p_T^2 p_1 \cdot p_2} - \frac{2 p_1 \cdot p_3 p_2^\mu p_3^\nu}{p_T^2 p_1 \cdot p_2} - \frac{2 p_2 \cdot p_3 p_3^\mu p_1^\nu}{p_T^2 p_1 \cdot p_2} + \frac{2 p_3^\mu p_3^\nu}{p_T^2}$$

Glover, van der Bij 88

slide: S.Jones

projectors onto form factors

Construct Projectors:

No Integrals

$$P_j^{\mu\nu} = \sum_{i=1}^{2} B_{ji}(\hat{s}, \hat{t}, m_H^2, d) T_i^{\mu\nu}$$

Same Basis as amplitude

Such that:

$$P_{1\mu\nu}\mathcal{M}^{\mu\nu} = F_1$$

$$P_{2\mu\nu}\mathcal{M}^{\mu\nu} = F_2$$

Separately calculate the contraction of each projector with $\mathcal{M}^{\mu\nu}$

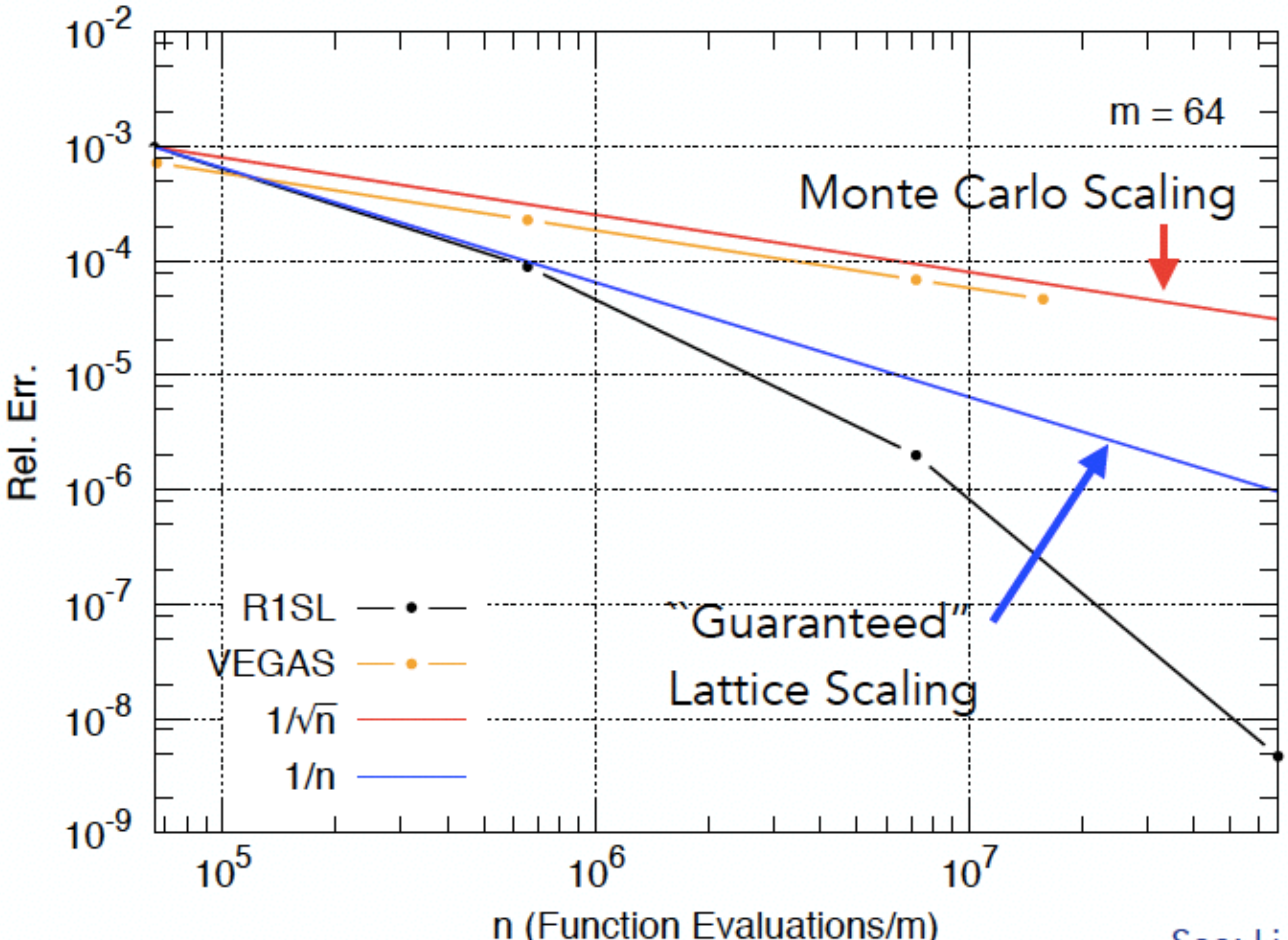
Projectors (CDR $D = 4 - 2\epsilon$):

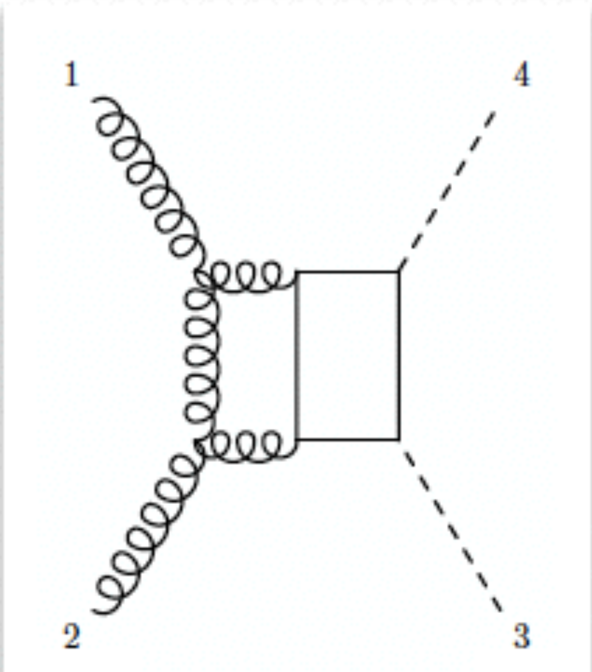
$$P_1^{\mu\nu} = \frac{1}{4}\frac{D-2}{D-3}T_1^{\mu\nu} - \frac{1}{4}\frac{D-4}{D-3}T_2^{\mu\nu}$$

$$P_2^{\mu\nu} = -\frac{1}{4}\frac{D-4}{D-3}T_1^{\mu\nu} + \frac{1}{4}\frac{D-2}{D-3}T_2^{\mu\nu}$$

slide: S.Jones

Example: Rel. Err. of one sector of sector decomposed loop integral





6 dimensional numerical integral

See: Li, Wang, Yan, Zhao 15

slide: S.Jones

pySecDec usage

slide: Stephan Jahn

$$\int_0^1 dx \int_0^1 dy (x+y)^{-2+\epsilon} = \frac{1}{\epsilon} + (1 - \log(2)) + O(\epsilon) \approx \frac{1}{\epsilon} + 0.306853 + O(\epsilon)$$

Step 1: write input files

generate_easy.py

```
from pySecDec import make_package

make_package(

name = 'easy',
integration_variables = ['x','y'],
regulators = ['eps'],

requested_orders = [0],
polynomials_to_decompose = ['(x+y)^(-2+eps)'],

)
```

integrate_easy.py

```
from pySecDec.integral_interface \
    import IntegralLibrary

# load c++ library
easy_integral = \
    IntegralLibrary('easy/easy_pylink.so')

# integrate
_, _, result = easy_integral()

# print result
print('Numerical Result:')
print(result)
```

Step 2: run pySecDec

```
$ python generate_easy.py && make -C easy && python integrate_easy.py
<skipped some output>
Numerical Result:
 + (1.00015897181235158e+00 +/- 4.03392522752491021e-03)*eps^-1 + (3.06903035514056399e-01 +/-
   ↪     2.82319349818329918e-03) + O(eps)
```