Progress in the Nested Soft-Collinear Subtraction Scheme

Raoul Röntsch

Les Houches Workshop

12 June 2019

Subtraction Methods at NNLO

- Basic idea: identify a function *S* which:
  - ➢ reproduces the matrix elements in the unresolved limits;
  - ➢ is (relatively) simple and can be integrated over the unresolved phase space.
- Subtract and add back:

$$\int |\mathcal{M}|^2 F_J d\phi_d = \int \left(|\mathcal{M}_J|^2 F_J - S\right) d\phi_4 + \int S d\phi_d$$

Divergent

Finite;
integrate in 4-dim.

Counterterm;
Explicit singularities

- Pros:
  - ✔ Local – better numerical stability.
  - ✔ No issues of cutoff or power corrections.
  - ✔ Historically, subtraction outperformed slicing at NLO.
- Cons:
  - ✗ Difficult to identify good subtraction function.
  - ✗ Highly non-trivial to integrate counterterm – singularities overlap.

The NNLO Revolution (continues?)

- Great phenomenological success for a variety of 2->2 processes across the LHC SM programme.
- BUT: None of the subtraction schemes are completely satisfactory (esp. compared to NLO):
  - Local
    - *Subtraction point-by-point in phase space.*
    - *Clear physical origins of singularities.*
    - *Avoid large numerical cancellations in intermediate steps.*
  - Analytic
    - *Poles cancel explicit – full control over singularity structures.*
    - *Improved numerical efficiency.*
  - Generic
    - *Accommodate arbitrary production processes at the LHC, including massive quarks.*
  - Minimal
    - *Clear origin of singularities.*
    - *Easier for others to implement.*

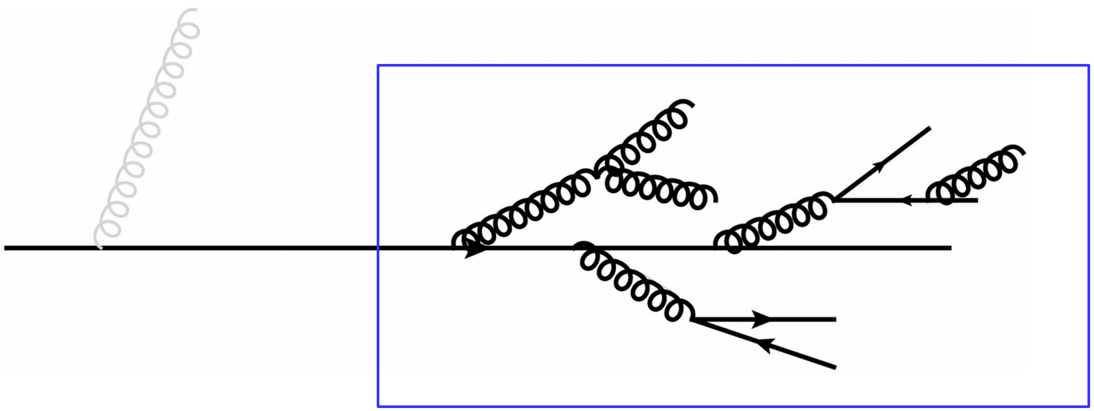
Nested Soft-Collinear Subtractions

[Caola, Melnikov, R.R. '17]

- Idea: extend FKS subtraction to NNLO.
  - Fully local.
- Difficulty: overlapping singularities in the integration of the counterterm.
- E.g. $q(p_1)\bar{q}(p_2) \to V + g(p_4) + g(p_5)$:
- IR singularities from
  - g_4 and g_5 soft (double soft)
  - g_4 or g_5 soft (single soft)
  - g_4 and g_5 collinear to either q or qb (triple collinear)
  - g_4 or g_5 collinear to either q or qb, or g_4 and g_5 collinear to each other (single collinear)
  - Singularities overlap (physical feature of QCD at NNLO).
  - Look at Feynman diagrams --> soft and collinear overlap!

Color coherence

- On-shell, gauge-invariant QCD scattering amplitudes : color coherence.
- Used in resummation & parton showers; not manifest in subtractions.
- Soft gluon cannot resolve details of collinear splittings; only sensitive to total color charge.



➡ No overlap between soft and collinear limits -- can be treated independently:

- Regularize soft singularities first, then collinear singularities.
- Energies and angles decouple.

Removing Soft Singularities

- Introduce energy ordering to remove trivial overlapping soft singularities.
- Regulate soft singularities as in FKS

$$\left\langle F_{LM}(1,2,4,5)\right\rangle = \left\langle \mathbb{S} F_{LM}(1,2,4,5)\right\rangle + \left\langle S_5 (I - \mathbb{S}) F_{LM}(1,2,4,5)\right\rangle + \left\langle (I - S_5)(I - \mathbb{S}) F_{LM}(1,2,4,5)\right\rangle.$$

- Analytic expressions for single and double soft counterterms.

  [Delto, Caola, Frellesvig, Melnikov ‘18]

- Soft subtracted term has only collinear divergences.
  - Remove these using sector decomposition, a la STRIPPER [Czakon ‘10, ‘11]

Phase-space partitioning

- Introduce **phase-space partitions**

$$1 = w^{14,15} + w^{24,25} + w^{14,25} + w^{15,24}.$$

with

$$C_{42} w^{14,15} = C_{52} w^{14,15} = 0 \quad \Rightarrow \quad w^{14,15} \text{ contains } C_{41},\ C_{51},\ C_{45}$$

$$C_{41} w^{24,25} = C_{51} w^{24,25} = 0 \qquad\quad w^{24,25} \text{ contains } C_{42},\ C_{52},\ C_{45}$$

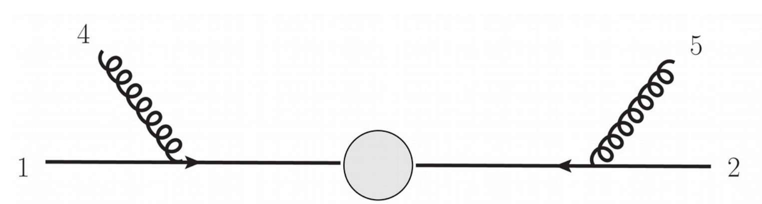
Triple collinear partition

and

$$C_{42} w^{14,25} = C_{51} w^{14,25} = C_{45} w^{14,25} = 0 \quad \Rightarrow \quad w^{14,25} \text{ contains } C_{41},\ C_{52}$$

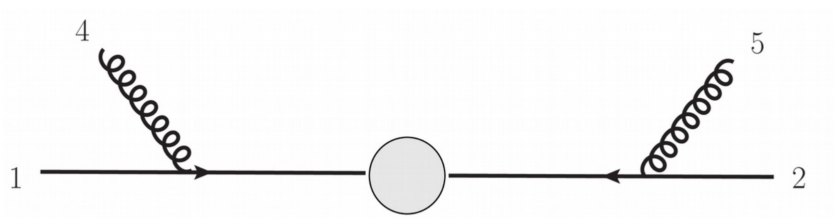
$$C_{41} w^{15,24} = C_{52} w^{15,24} = C_{45} w^{15,24} = 0 \qquad\quad w^{15,24} \text{ contains } C_{42},\ C_{51}$$

Double collinear partition



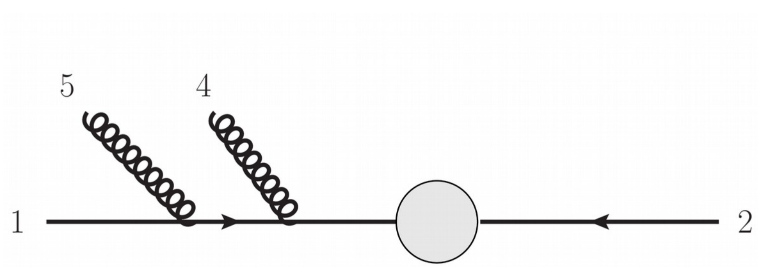
Phase-space partitioning

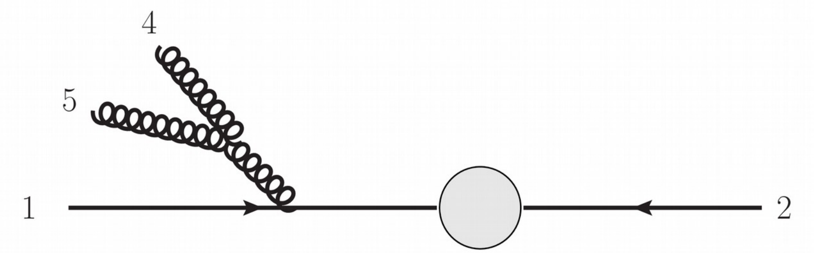
- Double collinear partition – large rapidity difference.



~ NLO x NLO ➡ simple

- Triple collinear partition – large/small rapidity difference.





Overlapping singularities remain – need one last step to separate these.

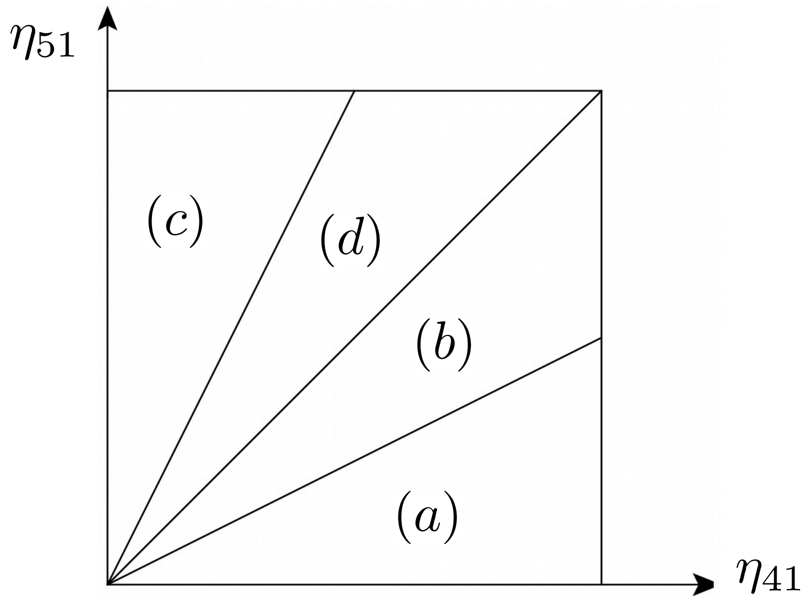
Sector Decomposition

- **Step 5: Sector decomposition:**

- Define angular ordering to separate singularities.

$\eta_{ij} = \rho_{ij}/2$

$$1 = \theta\left(\eta_{51} < \frac{\eta_{41}}{2}\right) + \theta\left(\frac{\eta_{41}}{2} < \eta_{51} < \eta_{41}\right) + \theta\left(\eta_{41} < \frac{\eta_{51}}{2}\right) + \theta\left(\frac{\eta_{51}}{2} < \eta_{41} < \eta_{51}\right) \equiv \theta^{(a)} + \theta^{(b)} + \theta^{(c)} + \theta^{(d)}.$$



- Thus the limits are

$\theta^{(a)} : C_{51}$, $\theta^{(c)} : C_{41}$ } Large rapidity difference

$\theta^{(b)} : C_{45}$, $\theta^{(d)} : C_{45}$ } Small rapidity difference

- Sectors *a,c* and *b,d* same to *4* ↔ *5*, but recall energy ordering.
- Implemented through angular phase space parametrization [Czakon '10].

Removing Collinear Singularities

- Starting from the soft subtracted term, subtract single and triple collinear configurations (using partitioning + sector decomposition).
- Analytic expressions for integrated triple collinear counterterms computed recently [Delto, Melnikov '19].
- All singularities removed through nested subtractions.
  - Follows naturally from separation of soft and collinear divergences.

Nested Soft-Collinear Subtractions

The method is therefore:

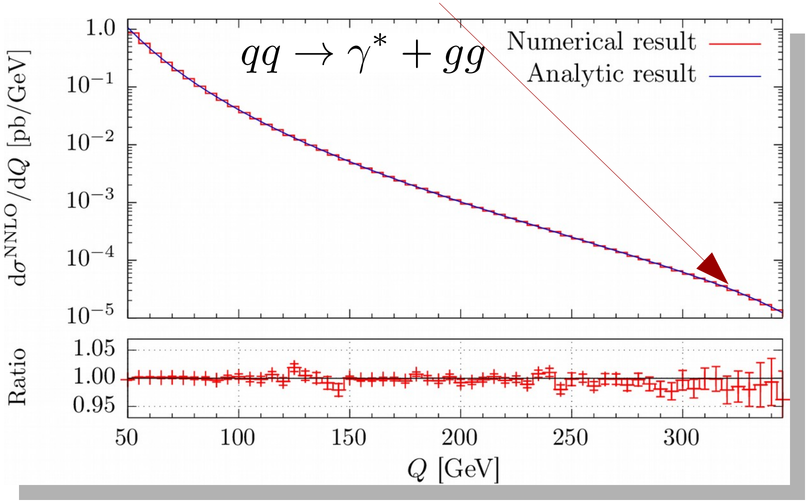
- Fully local.
- Fully analytic.
- Minimal – regulate only those physical singularities appearing in QCD, in each partition and sector.
- Flexible – it is not tied to any parametrization.
  - STRIPPER parametrization used at present but one could explore different parametrizations.

How does it do in practice?

Validation of Results

- Exhaustively tested against analytic results for
  - ✔ Drell-Yan production [Hamberg, Matsuura, van Neerven ‘89]
  - ✔ Higgs production [Anastasiou, Melnikov ‘04]

- Good control in extreme kinematic regions.



[Caola, Melnikov, R.R. ‘17]

- < per mille agreement for all NNLO *contributions*, including numerically tiny ones.

Channel	Color structures	Numerical result (nb)	Analytic result (nb)
$q_i\bar{q}_i \to gg$	–	8.351(1)	8.3516
$q_i\bar{q}_i \to q_j\bar{q}_j$	$C_F T_R n_{\mathrm{up}},\ C_F T_R n_{\mathrm{dn}}$	-2.1378(5)	-2.1382
	$C_F(C_A - 2C_F)$	$-4.8048(3)\cdot 10^{-2}$	$-4.8048\cdot 10^{-2}$
	$C_F T_R$	$5.441(7)\cdot 10^{-2}$	$5.438\cdot 10^{-2}$
$q_i q_j \to q_i q_j\ (i \neq -j)$	$C_F T_R$	0.4182(5)	0.4180
	$C_F(C_A - 2C_F)$	$-9.26(1)\cdot 10^{-4}$	$-9.26\cdot 10^{-4}$
$q_i g + g q_i$	–	-9.002(9)	-8.999
gg	–	1.0772(1)	1.0773

Table 1: Different contributions to the NNLO *coefficient* for on-shell Z production at the 13 TeV LHC with $\mu_R = \mu_F = 2m_Z$. All the color factors are included in the numerical results. The residual Monte-Carlo integration error is shown in brackets. See text for details.

[Caola, Melnikov, R.R. ‘19]

Reliable – able to compute NNLO corrections at arbitrary precision.

Validation of Results

Implies absolute control on physical results.

In 1 hour on a standard 1-core laptop:

- Higgs production, total cross section at one per mille:

$$\sigma_H^{\rm LO} = 17.03(0)\ {\rm pb}; \quad \sigma_H^{\rm NLO} = 30.25(1)\ {\rm pb}; \quad \sigma_H^{\rm NNLO} = 39.96(2)\ {\rm pb}.$$

- Drell-Yan production with symmetric cuts on leptons, cross sections at 2 per mille.

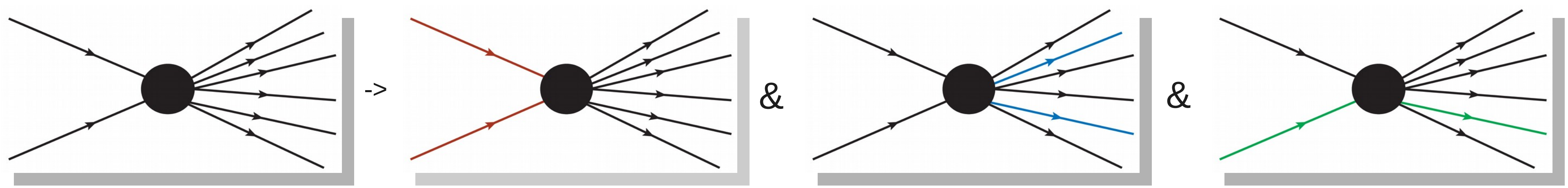
$$\sigma_{\rm DY}^{\rm LO} = 650.4 \pm 0.1\ {\rm pb}; \quad \sigma_{\rm DY}^{\rm NLO} = 700.2 \pm 0.3\ {\rm pb}; \quad \sigma_{\rm DY}^{\rm NNLO} = 734.8 \pm 1.4\ {\rm pb}.$$

- Very simple processes, but indicates that the method is efficient.

- Can it do 2->3???

Building Towards Generality

Split $pp \to n$ parton process into:



Initial-Initial color (color singlet production)

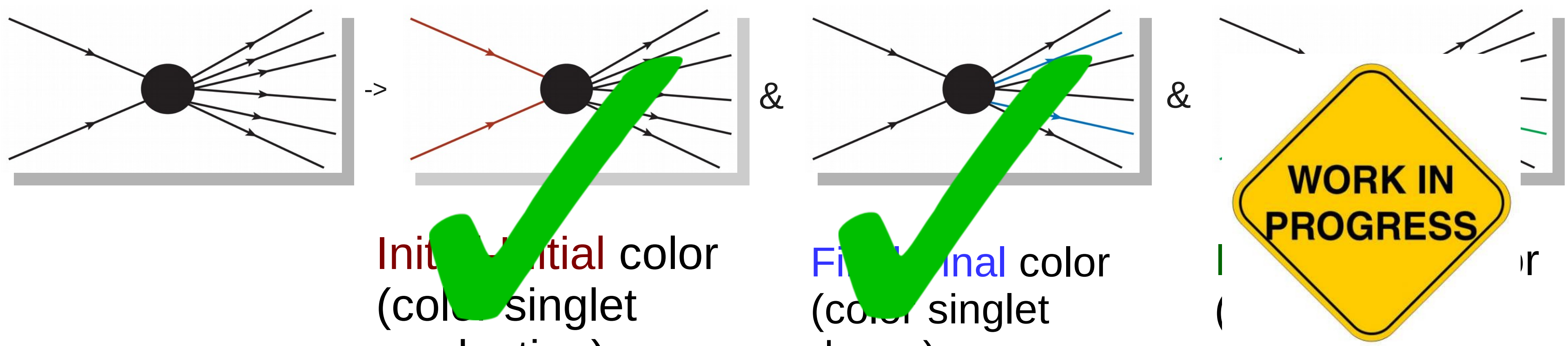
Final-Final color (color singlet decay)

Initial-Final color (deep inelastic scattering)

1. Consider color singlet production, color singlet decay, deep inelastic scattering in turn.
2. Compare against analytic results ➡ ***complete control*** on each block.

Building Towards Generality

Split $pp \rightarrow n$ parton process into:



Init itial color (col singlet production)

Fi inal color (co singlet decay)

I r (scattering)

[Caola, Melnikov, RR, 1902.02081]

[Caola, Delto, Melnikov, RR, 1906.xxxxx]

[Asteriadis, Caola, Melnikov, RR, 19yy.zzzzz]

Building Towards Generality

- Once the II, FF and IF are in place, these can be assembled for processes with 2->1 partons at Born level *(H+j, V+j, ...).*
- Going to 2->2 partons (e.g. dijet, H+2j) requires an understanding of non-trivial color-correlations.
- 2->3 partons (e.g. trijet) does not provide any new conceptual issues.
- At this stage, difficult to comment on issues like runtime, numerical stability, etc. for such high multiplicity processes.

Conclusions

- Despite success of IR subtraction schemes, ultimate scheme yet to be developed.
- Proposed **nested soft-collinear scheme:**
  - Fully local, fully analytic, minimal, flexible.
- Constructing a general subtraction framework for 2->2 & 2->3 partonic processes:
  - Initial-initial partons (color singlet production) ✓
  - Final-final partons (color singlet decay) ✓
  - Initial-final partons (DIS) 

  - Color correlations
- Stay tuned...