Quick Overview of N-Jettiness Subtractions

Frank Tackmann

Deutsches Elektronen-Synchrotron

June 13, 2019, Les Houches

Starting Point.

$$\sigma(X) = \int \mathrm{d}\mathcal{T}_N \frac{\mathrm{d}\sigma(X)}{\mathrm{d}\mathcal{T}_N} = \underbrace{\int^{\mathcal{T}_{\mathrm{cut}}} \mathrm{d}\mathcal{T}_N \frac{\mathrm{d}\sigma(X)}{\mathrm{d}\mathcal{T}_N}}_{\equiv\, \sigma(X,\, \mathcal{T}_{\mathrm{cut}})} + \int_{\mathcal{T}_{\mathrm{cut}}} \mathrm{d}\mathcal{T}_N \frac{\mathrm{d}\sigma(X)}{\mathrm{d}\mathcal{T}_N}$$

- $\sigma(X)$: generic N-jet cross section
  - At LO_N: $\sigma^{\mathrm{LO}}(X) = \int \mathrm{d}\Phi_N\, B_N(\Phi_N)\, X(\Phi_N)$
  - X: All defining Born-level measurements/cuts
  - Φ_N: Born-level phase-space
- $\mathcal{T}_N$: physical IR-safe N-jet resolution variable

  $$\mathcal{T}_N(\Phi_N) = 0 \qquad \mathcal{T}_N(\Phi_{\geq N+1}) > 0 \qquad \mathcal{T}_N(\Phi_{\geq N+1} \to \Phi_N) \to 0$$

- $\mathrm{d}\sigma(X)/\mathrm{d}\mathcal{T}_N$: differential $\mathcal{T}_N$ spectrum
  - At LO_N: $\dfrac{\mathrm{d}\sigma^{\mathrm{LO}}(X)}{\mathrm{d}\mathcal{T}_N} = \sigma^{\mathrm{LO}}(X)\, \delta(\mathcal{T}_N)$
  - $\mathcal{T}_N > 0$ defines an IR-safe physical N+1-jet cross section

Subtractions.

Add and subtract $\quad \sigma^{\rm sub}(\mathcal{T}_{\rm off}) = \sigma^{\rm sub}(\mathcal{T}_{\rm cut}) + \int_{\mathcal{T}_{\rm cut}}^{\mathcal{T}_{\rm off}} \mathrm{d}\mathcal{T}_N \frac{\mathrm{d}\sigma^{\rm sub}}{\mathrm{d}\mathcal{T}_N}$

$$\sigma = \sigma(\mathcal{T}_{\rm cut}) + \int_{\mathcal{T}_{\rm cut}} \mathrm{d}\mathcal{T}_N \frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}_N}$$

$$= \sigma^{\rm sub}(\mathcal{T}_{\rm off}) + \int_{\mathcal{T}_{\rm cut}} \mathrm{d}\mathcal{T}_N \left[\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}_N} - \frac{\mathrm{d}\sigma^{\rm sub}}{\mathrm{d}\mathcal{T}_N}\theta(\mathcal{T} < \mathcal{T}_{\rm off})\right] + \sigma(\mathcal{T}_{\rm cut}) - \sigma^{\rm sub}(\mathcal{T}_{\rm cut})$$

NNLO$_N$ NLO$_{N+1}$ neglect

- Subtractions $\sigma^{\rm sub}(\mathcal{T}_{\rm cut})$ and $\mathrm{d}\sigma^{\rm sub}/\mathrm{d}\mathcal{T}_N$ must reproduce singular limit of $\sigma(\mathcal{T}_{\rm cut})$ and $\mathrm{d}\sigma/\mathrm{d}\mathcal{T}_N$

Subtractions.

Add and subtract $\sigma^{\mathrm{sub}}(\mathcal{T}_{\mathrm{off}}) = \sigma^{\mathrm{sub}}(\mathcal{T}_{\mathrm{cut}}) + \int_{\mathcal{T}_{\mathrm{cut}}}^{\mathcal{T}_{\mathrm{off}}} \mathrm{d}\mathcal{T}_N \frac{\mathrm{d}\sigma^{\mathrm{sub}}}{\mathrm{d}\mathcal{T}_N}$

$$\sigma = \sigma(\mathcal{T}_{\mathrm{cut}}) + \int_{\mathcal{T}_{\mathrm{cut}}} \mathrm{d}\mathcal{T}_N \frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}_N}$$

$$= \sigma^{\mathrm{sub}}(\mathcal{T}_{\mathrm{off}}) + \int_{\mathcal{T}_{\mathrm{cut}}} \mathrm{d}\mathcal{T}_N \left[\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}_N} - \frac{\mathrm{d}\sigma^{\mathrm{sub}}}{\mathrm{d}\mathcal{T}_N}\theta(\mathcal{T} < \mathcal{T}_{\mathrm{off}})\right] + \sigma(\mathcal{T}_{\mathrm{cut}}) - \sigma^{\mathrm{sub}}(\mathcal{T}_{\mathrm{cut}})$$

$$= \underbrace{\sigma^{\mathrm{sub}}(\mathcal{T}_{\mathrm{cut}})}_{\text{NNLO}_N} + \underbrace{\int_{\mathcal{T}_{\mathrm{cut}}} \mathrm{d}\mathcal{T}_N \frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}_N}}_{\text{NLO}_{N+1}} + \underbrace{\sigma(\mathcal{T}_{\mathrm{cut}}) - \sigma^{\mathrm{sub}}(\mathcal{T}_{\mathrm{cut}})}_{\text{neglect}}$$

- Subtractions $\sigma^{\mathrm{sub}}(\mathcal{T}_{\mathrm{cut}})$ and $\mathrm{d}\sigma^{\mathrm{sub}}/\mathrm{d}\mathcal{T}_N$ must reproduce singular limit of $\sigma(\mathcal{T}_{\mathrm{cut}})$ and $\mathrm{d}\sigma/\mathrm{d}\mathcal{T}_N$
- $\mathcal{T}_{\mathrm{off}}$ is a priori arbitrary and exactly cancels
  - Determines $\mathcal{T}_N$ range over which subtraction acts *differentially* in $\mathcal{T}_N$
  - Setting $\mathcal{T}_{\mathrm{off}} = \mathcal{T}_{\mathrm{cut}}$ reduces it to a global subtraction (aka slicing)

Power Expansion.

Expand cross section in powers of $\tau_N \equiv \dfrac{\mathcal{T}_N}{Q}$ and $\tau_{\mathrm{cut}} \equiv \dfrac{\mathcal{T}_{\mathrm{cut}}}{Q}$

(where Q is a typical hard scale whose precise choice is irrelevant for now)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau_N} = \frac{\mathrm{d}\sigma^{(0)}}{\mathrm{d}\tau_N} + \frac{\mathrm{d}\sigma^{(2)}}{\mathrm{d}\tau_N} + \frac{\mathrm{d}\sigma^{(4)}}{\mathrm{d}\tau_N} + \cdots$$

$$\sigma(\tau_{\mathrm{cut}}) = \sigma^{(0)}(\tau_{\mathrm{cut}}) + \sigma^{(2)}(\tau_{\mathrm{cut}}) + \sigma^{(4)}(\tau_{\mathrm{cut}}) + \cdots$$

- Leading-power (singular) terms

$$\frac{\mathrm{d}\sigma^{\mathrm{sing}}}{\mathrm{d}\tau_N} \equiv \frac{\mathrm{d}\sigma^{(0)}}{\mathrm{d}\tau_N} \sim \delta(\tau_N) + \left[\frac{\ln^{n-1}\tau_N}{\tau_N}\right]_+$$

$$\sigma^{\mathrm{sing}}(\tau_{\mathrm{cut}}) \equiv \sigma^{(0)}(\tau_{\mathrm{cut}}) \sim \ln^n \tau_{\mathrm{cut}}$$

- Subleading-power (nonsingular) terms

$$\tau_N \frac{\mathrm{d}\sigma^{(2k)}}{\mathrm{d}\tau_N} \sim \mathcal{O}(\tau_N^k) \qquad \sigma^{(2k)}(\tau_{\mathrm{cut}}) \sim \mathcal{O}(\tau_{\mathrm{cut}}^k)$$

Putting Everything Together.

$$\sigma = \underbrace{\sigma^{\rm sub}(\tau_{\rm cut})}_{\text{NNLO}_N} + \underbrace{\int_{\tau_{\rm cut}} d\tau_N \frac{d\sigma}{d\tau_N}}_{\text{NLO}_{N+1}} + \underbrace{\Delta\sigma(\tau_{\rm cut})}_{\text{neglect}}$$

Subtractions have to satisfy

$$\sigma^{\rm sub}(\tau_{\rm cut}) = \sigma^{(0)}(\tau_{\rm cut})\,[1 + \mathcal{O}(\tau_{\rm cut})]$$

such that neglecting $\Delta\sigma(\tau_{\rm cut})$ only misses $\mathcal{O}(\tau_{\rm cut})$ power-suppressed terms

$$\Delta\sigma(\tau_{\rm cut}) = \sigma(\tau_{\rm cut}) - \sigma^{\rm sub}(\tau_{\rm cut}) = \sigma^{(2)}(\tau_{\rm cut}) + \cdots \sim \mathcal{O}(\tau_{\rm cut})$$

Tradeoff: Lowering $\tau_{\rm cut}$ ...

- ... reduces size of missing power corrections $\Delta\sigma(\tau_{\rm cut})$
- ... increases numerical cancellations between first two terms
  - Requires numerically more precise calculation of $d\sigma/d\tau_N$ in a region where the N+1-jet NLO calculation quickly becomes much less stable
  - Computational cost increases substantially

N-Jettiness Event Shape.

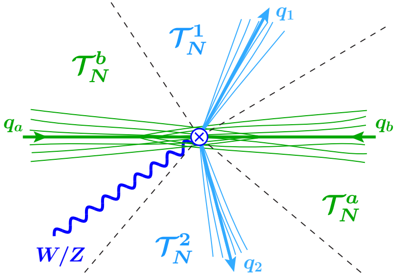
[Stewart, FT, Waalewijn, '10]

$$\mathcal{T}_N = \sum_k \min\Big\{ \frac{2q_a \cdot p_k}{Q_a}, \frac{2q_b \cdot p_k}{Q_b}, \frac{2q_1 \cdot p_k}{Q_1}, \frac{2q_2 \cdot p_k}{Q_2}, \ldots, \frac{2q_N \cdot p_k}{Q_N} \Big\}$$

$$\equiv \mathcal{T}_N^a + \mathcal{T}_N^b + \mathcal{T}_N^1 + \cdots + \mathcal{T}_N^N$$

- Partitions phase space into N jet regions and 2 beam regions
- $Q_{a,b}$, Q_j determine distance measure
  - Geometric measures: $Q_i = 2\rho_i E_i$
- Massless born reference momenta q_i

$$q_{a,b} = x_{a,b} \frac{E_{\rm cm}}{2}(1, \pm\hat{z}) \,, \;\; q_j = E_j(1, \vec{n}_j)$$



Their choice corresponds to choosing an (IR-safe) Born projection

- Does not affect leading-power structure and resummation
- Part of full N-jettiness definition and does affect power-suppressed terms

N-Jettiness Factorization Theorem.

[Stewart, FT, Waalewijn, '09, '10]

$$\frac{\mathrm{d}\sigma^{\mathrm{sing}}(X)}{\mathrm{d}\mathcal{T}_N} = \int \mathrm{d}\Phi_N \, \frac{\mathrm{d}\sigma^{\mathrm{sing}}(\Phi_N)}{\mathrm{d}\mathcal{T}_N} \, X(\Phi_N)$$

$$\frac{\mathrm{d}\sigma^{\mathrm{sing}}(\Phi_N)}{\mathrm{d}\mathcal{T}_N} = \int \mathrm{d}t_a \, B_a(t_a, x_a, \mu) \int \mathrm{d}t_b \, B_b(t_b, x_b, \mu) \Bigl[\prod_{i=1}^{N} \int \mathrm{d}s_i \, J_i(s_i, \mu) \Bigr]$$
$$\times \vec{C}^\dagger(\Phi_N, \mu) \, \widehat{S}_\kappa\Bigl(\mathcal{T}_N - \frac{t_a}{Q_a} - \frac{t_b}{Q_b} - \sum_{i=1}^{N} \frac{s_i}{Q_i}, \{\hat{q}_i\}, \mu\Bigr) \vec{C}(\Phi_N, \mu)$$

- All functions are IR finite and have an operator definition in SCET
- To obtain subtraction coefficients simply FO expand and collect terms
- Simplifying features of N-jettiness
  - No dependence on jet algorithm (jet clustering, jet radius, etc.)
  - No recoil effects from soft radiation
  - No additional $\vec{p}_T$ dependence or convolutions, no rapidity divergences
  - Overlap between soft and collinear contributions vanishes in pure dim. reg.

Pros and Cons.

Key advantages

- All IR-singular contributions are projected onto physical observable $\mathcal{T}_N$
  - Can be computed from fact. theorem for singular limit of cross section
  - Simpler structure and fewer subtraction terms
  - Also allows computing power corrections
- Nonsingular contributions are directly given in terms of existing lower-order Born+1-jet calculations

Potential drawbacks

- Subtractions are nonlocal (i.e. not point-by-point in real emission phase space)
  - Phase-space slicing in $\mathcal{T}_N$ = global (maximally nonlocal) subtraction
- In practice, it is a question of numerical stability whether this is a disadvantage or not
  - Naively expect larger numerical cancellations (since they happen later)
  - Most relevant is numerical stability of real-virtual and double-real matrix elements in deep unresolved limit which are always needed regardless of subtraction method